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Roll No.

BE–102

B. E. (First Semester) EXAMINATION, Dec., 2010

(Grading System)

(Common for all Branches)

ENGINEERING MATHEMATICS – I

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (D Grade)

Note : Attempt all questions. All questions carry equal marks.

1. (a) State and prove Taylor's theorem and expand $\log_e x$ in powers of $(x - 1)$.

   (b) If :

$$u = \begin{bmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{bmatrix}$$

   then evaluate :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

Or

   (a) Find the radius of curvature at the point 't' of the curve cycloid :

$$x = a(t + \sin t),\ y = a(1 - \cos t)$$

P. T. O.

(b) Discuss the maxima and minima of the function :

$$x^3 + y^3 - 3x - 12y + 20$$

2. (a) Prove that :

$$\int_0^1 (1 - x^n)^{1/n}\, dx = \frac{1}{n} \cdot \frac{\left\{\left\lceil \frac{1}{n} \right\}^2}{2 \cdot \left\lceil \frac{2}{n} \right.}$$

(b) Evaluate :

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\, dy}{1 + x^2 + y^2}$$

Or

(a) Find the area lying between the parabola :

$$y = 4x - x^2$$

and the line $y = x$.

(b) Evaluate as limit of sums :

$$\int_1^3 (x^2 + x)\, dx$$

3. (a) Solve :

$$(2x - b)\, p = y - ay\, p^2$$

where $p \equiv \frac{dy}{dx}$.

(b) Solve :

$$\{(D - 1)^2 (D - 3)^3\}\, y = e^{3x}$$

Or

(a) Solve :

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 20y = (x + 1)^2$$

(b) Solve :

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

4. (a) Find the rank of the matrix by reducing it to normal form :

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

(b) Show that the system of equations :

$$3x + 3y + 2z = 1$$

$$x + 2y = 4$$

$$10y + 3z = -2$$

$$2x - 3y - z = 5$$

are consistent and hence solve it.

Or

(a) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(b) State and prove Cayley-Hamilton theorem.

5. (a) Prove that a tree with n-vertices has $(n - 1)$ number of edges in it.

(b) Define the following :

(i) Walk

(ii) Path

(iii) Circuit

(iv) Union and Intersection of two graphs

Or

(a) Define principle of duality in Boolean Algebra and prove that :

$$(a')' = a, \forall\, a \in B$$

(b) Find out the disjunctive and conjunctive normal form of the polynomial :

$$F(x, y, z) = [x + (x' + y')'] \cdot [x + (y' \cdot z')']$$