Total No. of Questions :5] *[Total No. of Printed Pages :4*

Roll No ....

BE - 301

B.E. III Semester

Examination, December 2013

Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70

Note: i) Solve all the questions.
ii) All questions carry equal marks
iii) One full question should be solved at one place.

1. a) Find a Fourier series to represent $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Also deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$$

b) Obtain a half range cosine series for

$$f(x) = \begin{cases} kx & ,0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l \end{cases}$$

OR

a) Find the Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate $\int_{-\infty}^{\infty} \frac{\sin s \cos sx}{s} ds$.

b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

2. a) Find the Laplace transform of

i) $f(t) = \frac{1 - \cos 2t}{t}$

ii) $f(t) = t^3 e^{-3t}$

b) By convolution theorem, evaluate

$$L^{-1}\left\{\frac{S^2}{(S^2 + a^2)(S^2 + b^2)}\right\}$$

OR

a) Find the inverse Laplace transform of

i) $\bar{f}(s) = \log \frac{s+1}{s-1}$

ii) $\bar{f}(s) = \frac{1}{s^3(s^2+1)}$

b) Use Laplace transform method to solve the differential equation:

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t$$

with $y(0) = 2,\ y'(0) = -1$.

3. a) Solve the differential equation

$$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3.$$

b) Solve by series method:

$$\left(x - x^2\right)\frac{d^2y}{dx^2} + (1-5x)\frac{dy}{dx} - 4y = 0.$$

OR

a) Solve

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin x^2.$$

b) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$$

4. a) Solve the partial differential equations:

i) $y^2zp + x^2zq = y^2x$

ii) $\left(p^2 + q^2\right)y = qz$

b) Solve $\left(D^2 - DD'\right)Z = \sin x \cos 2y$

OR

a) Solve $\left(D^2 + DD' + D' - 1\right)Z = \sin(x + 2y)$.

b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement.

5. a) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (–1, –1, 2).

b) Show that the vector field $\overline{F} = \nabla\left(x^3 + y^3 + z^3 - 3xyz\right)$ is irrotational.

OR

a) Evaluate $\int_s F.ds$ where $F = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

b) By using Stokes theorem, evaluate: $\int_c (yzdx + zxdy + xzdz)$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

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