

M. E./M. Tech. (First Semester)
EXAMINATION, Dec., 2010

ADVANCED COMPUTATIONAL MATHEMATICS

Time : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 40

Note : Attempt any five questions. All questions carry equal marks.

1. (a) Show that the vectors $(1, 2, 0)$, $(0, 3, 0)$ and $(-1, 0, 1)$ are linearly dependent in the vectors space V_3 of real numbers \mathbf{R} .

- (b) Let $T: \mathbf{R}^2 \rightarrow \mathbf{R}^3$ be defined by :

$$T(x, y) = \{-x + 2y, y, -3x + 3y\}.$$

Find the matrix representation of T with respect to the base $A = \{(1, 2), (-2, 1)\}$ and $B = \{(-1, 0, 2), (1, 2, 3), (1, -1, -1)\}$ of \mathbf{R}^2 and \mathbf{R}^3 respectively.

2. (a) Give Mathematical formula for $\{He_n(x)\}$ [Hermite polynomial] and show that :

$$He_3(x) = x^3 - 3x.$$

Also write weight function for Hermite polynomial to be orthogonal.

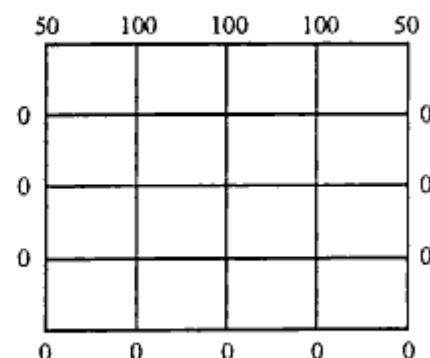
- (b) A string is stretched and fastened to two points l apart. Motion started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which is released at time $t = 0$. Show that the displacement of any point at distance x from one end at time t is given by :

$$y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi c t}{l}.$$

3. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to initial condition $u = \sin \pi x$ at $t = 0$ if $0 \leq x \leq 1$ and $u = 0$ at $x = 0$ and $x = 1$ for $t > 0$, by the Gauss-Seid iterative method.

- (b) Write the properties of DFT, WFT and Haar transform.

4. (a) Solve Laplace's equation for the following figure :



The boundary values being as indicated :

- (b) A can hit a target 4 times in 5 shots; B 3 times in 4 shots; C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit ?
5. (a) Fit a Poisson distribution to the following and calculate theoretical frequencies :
- | | | | | | | |
|-----|---|-----|----|----|---|---|
| x | : | 0 | 1 | 2 | 3 | 4 |
| f | : | 122 | 60 | 15 | 2 | 1 |
- (b) Write short notes on the following :
- Concept of estimation theory
 - Theory of hypotheses.
6. (a) Show that every stochastic process $\{x_t, t = 0, 1, 2, \dots\}$ with independent increment is a Markov process.
- (b) Obtain the distribution of the number in the system in steady state for (M/M/1 : ∞ /FCFS) model by considering it as birth and death process.
7. (a) Customers arrive at a one man barber shop according to Poisson process with a mean interarrival time of 12 minutes. Customer spend an average of 10 min, in the barber's shop :
- What is the expected number of customers in the barber shop and in the queue ?
 - How much time can a customer expect to spend in the barber's shop ?
 - What is the average time customer spends in the queue ?
- (b) What are fuzzy sets ? Describe the importance of fuzzy sets. Compare it with classical sets.
8. (a) Write five transfer functions and their MATLAB functions.
- (b) Define Heaviside's unit function and error function and where they are used.