6. a) A supermarket has a single cashier. During the peak hours customers arrive at a rate of 20 customers per hour. The average number of customer that can be processed by the cashier is 24 per hours calculate the following :

   i) Probability that the cashier is idle.

   ii) Average number of customer in the queuing system.

   iii) Queue size.

   iv) Average time a customer spends in the system.

   b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

7. a) Let A and B be two fuzzy sets defined on a universal set X prove that : $|A| + |B| = |A \cup B| + |A \cap B|$.

   b) Explain different functions which MATLAB provides in fuzzy tool box.

8. a) Define the following :

   i) Reliability

   ii) Failure rate

   iii) Hazard rate

   b) Explain :

   i) Decision theory

   ii) Goal programming

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Roll No .....

MEDC/MEMT/MEVD/MEIC/MEPE/ MEPS/MTPS/MTPA/MEHP-101

M.E/M.Tech., I Semester

Examination, December 2015

Advanced Mathematics

Time : Three Hours

Maximum Marks : 70

Note : i) Attempt any five questions.

ii) All questions carry equal marks.

iii) Normal Distribution table allowed.

1. a) Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

   b) Write short notes on the following :

   i) Wavelet transform

   ii) Haar transforms

2. a) Solve the Poisson's equation, $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y$, $x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1.

b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form

$y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$.

Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right).$$

3. a) A random variable X has the following probability function :

X	:	0	1	2	3	4	5	6	7
P(X)	:	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find the value of k

ii) Evaluate $P(X < 6)$, $P(X \geq 6)$

iii) $P(0 < X < 5)$

b) Write short notes on :

i) Stochastic process

ii) Markov process

4. a) In a bombing action there is 50% chance that any bomb will strike the target. Two direct hits are needed to destroy the target completely. How many bombs are required to be dropped to give a 99% chance or better of completely destroying the target?

b) Let $\{x_n, n \geq 0\}$ be a Markov chain having state process

$S = \{1, 2, 3, 4\}$ and transition matrix $P = \begin{vmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{vmatrix}$

5. a) Explain the following :

i) Elementary concept of estimation

ii) Theory of hypothesis

b) Patients arrive at a clinic according to a Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patients is exponential with mean rate 20 per hour then,

i) Find the effective arrival rate at the clinic.

ii) What is the probability that an arriving patients will not wait?

iii) What is the expected waiting time until a patient is discharged from the clinic?