Roll No ....

MMTP/MMCM/MMIE/MMMD/MMPD-101

M.E./M.Tech., I Semester

Examination, June 2014

Advance Mathematics

Time : Three Hours

Maximum Marks : 70

Note : Attempt any five questions. All questions carry equal marks.

1. a) Define Vector space and show that the set of all matrices of order 2×2 is vector space with respect to matrix addition and scalar multiplication.

   b) Show that the mapping $f: V_3(R) \to V_2(R)$ defined by

   f (a, b, c) = (a, b) is linear transformation. What is Kernel of this transformation?

2. a) Solve the Poisson's Equation

   $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -10(x^2 + y^2 + 10)$ over the square with sides

   x = 0 = y, x = 3 = y with u (x, y) = 0 on the boundary and mesh length h = 1.

   b) Find the solution of one dimensional heat equation by variable separable method.

3. a) A die was thrown 9000 times and throw of 5 or 6 was obtained 3240 times. On assumption of random throwing, do the data indicate an unbiased die?

b) Show that the mean deviation from the mean of normal distribution is $\frac{4}{5}$ of standard deviation.

4. a) Obtain the steady state difference equation for the queuing model (M/M/1) : (∞/FCFS).

b) Suppose a Markov Process has two states E_1 and E_2 and transition probabilities given by transition matrix (TPM);

$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ by means of separate chance device we choose E, with probability that the process is in state E after the first step.

5. a) Define the following:

i) Ritz Method

ii) Galerkin's Method

b) Use Rayleigh Method to solve the equation:

$$\frac{d^2y}{dx^2} + y = x \quad y(0)=0,\ y(1)=1.$$

6. a) Find the Fourier Transform of

$$f(x) = \begin{cases} 1, & for |x| < 1 \\ 0, & for |x| > 1 \end{cases}$$ Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

b) Prove that $H_{n+1}(x) = 2x\, H_n(x) - 2n\, H_{n-1}(x)$.

7. a) Find the mean and variance of Poisson distribution.

b) Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of the phone call is assumed to be distributed exponentially with mean 3 minutes:

i) What is the probability that a person arriving at the booth will have to wait in the queue?

ii) What is the average length of the queue that forms time to time?

8. Define each of the following:

i) Heaviside Unit Step Function.

ii) Theory of testing hypothesis

iii) Theory of estimators

iv) Discrete Random Variable

v) Matrix of Transition Probabilities.

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