1. a) Find the real root of the equation $x\log_{10} x = 1.2$ by bisection method correct to four decimal places.

Solution: Suppose $f(x) = x\log_{10} x - 1.2$

Taking, $f(2) = 2\log_{10}(2) - 1.2 = -0.59794(-ve)$

And $f(3) = 3\log_{10}(3) - 1.2 = 0.23136(ve)$

Therefore, the root lies between 2 and 3.

1st Approximation:

$$x_1 = \frac{2+3}{2} = 2.5$$

and $f(2.5) = 2.5\log_{10}(2.5) - 1.2 = -0.20514(-ve)$

$\therefore$ Roots lie between 2.5 and 3.

2nd Approximation

$$x_2 = \frac{2.5+3}{2} = 2.75$$

and $f(2.75) = 2.75\log_{10}(2.75) - 1.2 = 0.00816(+ve)$

$\therefore$ Roots lie between 2.5 and 2.75.

3rd Approximation

$$x_3 = \frac{2.5+2.75}{2} = 2.625$$

and $f(2.625) = 2.625\log_{10}(2.625) - 1.2 = -0.09978(-ve)$

$\therefore$ Roots lie between 2.625 and 2.75.

4th Approximation

$$x_4 = \frac{2.625+2.75}{2} = 2.6875$$

and $f(2.6875) = 2.6875\log_{10}(2.6875) - 1.2 = -0.04612(-ve)$

$\therefore$ Roots lie between 2.6875 and 2.75.

5th Approximation

$$x_5 = \frac{2.6875+2.75}{2} = 2.71875$$

and $f(2.71875) = 2.71875\log_{10}(2.71875) - 1.2 = -0.01905(-ve)$

$\therefore$ Roots lie between 2.71875 and 2.75.

6th Approximation

$$x_6 = \frac{2.71875+2.75}{2} = 2.734375$$

and $f(2.734375) = 2.734375\log_{10}(2.734375) - 1.2 = -0.005466(-ve)$

$\therefore$ Roots lie between 2.71875 and 2.75.

7th Approximation

$$x_7 = \frac{2.734375+2.75}{2} = 2.74218$$

and $f(2.74218) = 2.74218\log_{10}(2.74218) - 1.2 = -0.001337(-ve)$

$\therefore$ Roots lie between 2.74218 and 2.75.

8th Approximation

$$x_8 = \frac{2.74218 + 2.75}{2} = 2.74609$$

and $f(2.74609) = 2.74609\log_{10}(2.74609) - 1.2 = -0.004750(-ve)$

$\therefore$ Roots lie between 2.74609 and 2.75.

9th Approximation

$$x_9 = \frac{2.74609 + 2.75}{2} = 2.748045$$

and $f(2.748045) = 2.748045\log_{10}(2.748045) - 1.2 = -0.006457(-ve)$

$\therefore$ Roots lie between 2.748045 and 2.75.

10th Approximation

$$x_{10} = \frac{2.748045 + 2.75}{2} = 2.74902$$

and $f(2.74902) = 2.74902\log_{10}(2.74902) - 1.2 = -0.007308(-ve)$

$\therefore$ Roots lie between 2.74902 and 2.75.

11th Approximation

$$x_{11} = \frac{2.74902 + 2.75}{2} = 2.74951$$

and $f(2.74951) = 2.74951\log_{10}(2.74951) - 1.2 = -0.00773(-ve)$

$\therefore$ Roots lie between 2.74951 and 2.75.

12th Approximation

$$x_{12} = \frac{2.74951 + 2.75}{2} = 2.749755$$

and $f(2.749755) = 2.749755\log_{10}(2.749755) - 1.2 = -0.00795(-ve)$

$\therefore$ Roots lie between 2.749755 and 2.75.

13th Approximation

$$x_{12} = \frac{2.74951 + 2.75}{2} = 2.749755$$

and $f(2.749755) = 2.749755\log_{10}(2.749755) - 1.2 = -0.00795(-ve)$

$\therefore$ Roots lie between 2.749755 and 2.75.

Therefore, required root at three decimal places is 2.278. **Answer.**

(b) Find a real root of the equation $x = e^{-x}$ using the Newton-Rapshon method.

Solution: Given equation is $xe^x - 1 = 0$

Suppose $f(x) = xe^x - 1$

Taking $x = 0,\quad f(0) = (0)e^0 - 1 = -1(-ve)$

and $x = 1,\quad f(1) = (1)e^1 - 1 = 1.71828(+ve)$

Therefore a root lies between 0 and 1.

Now, $f'(x) = xe^x + e^x = (x+1)e^x$

Taking $x_0 = \dfrac{0+1}{2} = 0.5 \in (0,1)$, such that $f'(0.5) \neq 0$

1st Approximation: http://www.rgpvonline.com

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow \quad x_1 = x_0 - \frac{x_0 e^{x_0} - 1}{(x_0+1)e^{x_0}} = 0.5 - \frac{(0.5)e^{0.5} - 1}{(0.5+1)e^{0.5}} = 0.571020$$

2nd Approximation

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow \quad x_2 = x_1 - \frac{x_1 e^{x_1} - 1}{(x_1+1)e^{x_1}} = 0.57102 - \frac{(0.57102)e^{0.57102} - 1}{(0.57102+1)e^{0.57102}} = 0.56715$$

3rd Approximation

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\Rightarrow \quad x_3 = x_2 - \frac{x_2 e^{x_2} - 1}{(x_2+1)e^{x_2}} = 0.56715 - \frac{(0.56715)e^{0.56715} - 1}{(0.56715+1)e^{0.56715}} = 0.56714$$

Therefore, required root at four decimal places is 0.56714. **Answer.**

2. (a) Solve the following system of equations by Crout's method.

$x+y+z=3$, $2x-y+3z=16$ **and** $3x+y-z=-3$

Solution: The given system, in the matrix form, is

$$\begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 3\\ 3 & 1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 16\\ -3\end{bmatrix}$$

$$\Rightarrow \qquad AX=B \qquad \ldots\ldots (1)$$

By the assumption of LU method, we have http://www.rgpvonline.com

$$A=LU$$

$$\therefore \qquad \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 3\\ 3 & 1 & -1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ l_{21} & 1 & 0\\ l_{31} & l_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 3\\ 3 & 1 & -1\end{bmatrix}=\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23}\\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33}\end{bmatrix}$$

On equating the corresponding elements of equal matrices,

$$u_{11}=1 \qquad u_{12}=1 \qquad u_{13}=1$$

$$l_{21}u_{11}=2 \quad \Rightarrow \quad l_{21}=\frac{2}{1}=2$$

$$l_{21}u_{12}+u_{22}=-1 \quad \Rightarrow \quad 2(1)+u_{22}=-1 \quad \Rightarrow \quad u_{22}=-3$$

$$l_{21}u_{13}+u_{23}=3 \quad \Rightarrow \quad 2(1)+u_{23}=3 \quad \Rightarrow \quad u_{23}=1$$

$$l_{31}u_{11}=3 \quad \Rightarrow \quad l_{31}(1)=3 \quad \Rightarrow \quad l_{31}=3$$

$$l_{31}u_{12}+l_{32}u_{22}=1 \quad \Rightarrow \quad 3(1)+l_{32}(-3)=1 \quad \Rightarrow \quad l_{32}=\frac{2}{3}$$

$$l_{31}u_{13}+l_{32}u_{23}+u_{33}=-1 \quad \Rightarrow \quad 3(1)+\frac{2}{3}(1)+u_{33}=-1 \quad \Rightarrow \quad u_{33}=-1-3-\frac{2}{3}=-\frac{14}{3}$$

Therefore the value of L and U is,

$$L=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2/3 & 1\end{bmatrix} \text{ and } U=\begin{bmatrix}1 & 1 & 1\\ 0 & -3 & 1\\ 0 & 0 & -14/3\end{bmatrix}$$

Putting $A=LU$ in equation (1), we get

$$(LU)X=B$$

$$\Rightarrow \qquad L(UX)=B \qquad \ldots\ldots (2)$$

Taking $UX=Y$ so that (2) becomes

$$LY=B$$

$$\Rightarrow \qquad \begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2/3 & 1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\begin{bmatrix}3\\ 16\\ -3\end{bmatrix}$$

Equating the elements on both sides, we get

$y_1 = 3$ | $2y_1 + y_2 = 16$ | $3y_1 + \frac{2}{3}y_2 + y_3 = -3$

$\Rightarrow y_2 = 16 - 2(3) = 10$ $\quad\Rightarrow 3(3) + \frac{2}{3}(10) + y_3 = -3$ i.e. $y_3 = -3 - 9 - \frac{20}{3} = -\frac{56}{3}$

Therefore $Y = \begin{bmatrix} 3 \\ 10 \\ -56/3 \end{bmatrix}$

Now, $UX = Y$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -14/3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ -56/3 \end{bmatrix}$$

Equating the elements on both sides, we get

$x + y + z = 3$

$\Rightarrow x - 2 + 4 = 3$

$\Rightarrow x = 1$

$-3y + z = 10$

$\Rightarrow -3y + 4 = 10$

$\Rightarrow y = -2$

$-\frac{14}{3}z = -\frac{56}{3}$

$\Rightarrow z = 4$

Hence, $\boxed{x = 1,\ y = -2 \text{ and } z = 4}$ **Answer**

(b) Find $\frac{dy}{dx}$ at $x = 0.1$, from the following table

x	0.1	0.2	0.3	0.4
$f(x)$	0.9975	0.9900	0.9776	0.9604

Solution: Forward difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
$0.1 = x_0$	$0.9975 = y_0$			
		−0.0075		
0.2	0.9900		**−0.0049**	
		−0.0124		**0.0001**
0.3	0.9776		−0.0048	
		−0.0172		
0.4	0.9604			

Here, $h = 0.1$, $x_0 = 0.1$ and $x = 0.1$

Therefore, $p = \frac{x - x_0}{h} = \frac{0.1 - 0.1}{0.1} = 0$

By Newton's forward formula

$$\left(\frac{dy}{dx}\right)_{x=0.1} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \ldots\right]$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0.1} = \frac{1}{0.1}\left[-0.0075 - \frac{1}{2}(-0.0049) + \frac{1}{3}(0.0001)\right] = -0.050167$$

Hence, $\boxed{\left(\frac{dy}{dx}\right)_{x=0.1} = -0.5016}$ **Answer**

3. a) Using Euler's method compute y(0.04) for the differential equation.

$$\frac{dy}{dx} = -y, \quad y(0)=1, \quad \textbf{Take } h = 0.01$$

Solution: Given differential equation is,

$$\frac{dy}{dx} = -y = f(x, y) \qquad \ldots (1)$$

with initial condition, $y_0 = 1$ at $x_0 = 0$. And $x = 0.04$, $h = 0.01$ (Given)

Taking, $n = \dfrac{x - x_0}{h} = \dfrac{0.04 - 0}{0.01} = 4$

$\therefore \quad x_1 = x_0 + h = 0.01$, $x_2 = x_0 + 2h = 0.02$, $x_3 = x_0 + 3h = 0.03$ and $x_4 = x_0 + 4h = 0.04$

First Approximation:

$$y_1 = y_0 + h\,f(x_0, y_0)$$

$$\Rightarrow \quad y_1 = y_0 + h[-y_0]$$

$$\Rightarrow \quad y_1 = 1 + 0.01[-1] = 0.99 \qquad \text{at } x_1 = 0.01$$

Second Approximation: http://www.rgpvonline.com

$$y_2 = y_1 + h\,f(x_1, y_1)$$

$$\Rightarrow \quad y_2 = y_1 + h[-y_1]$$

$$\Rightarrow \quad y_2 = 0.99 + 0.01[-0.99] = 0.9801 \qquad \text{at } x_2 = 0.02$$

Third Approximation:

$$y_3 = y_2 + h\,f(x_2, y_2)$$

$$\Rightarrow \quad y_3 = y_2 + h[-y_2]$$

$$\Rightarrow \quad y_3 = 0.9801 + 0.01[-0.9801] = 0.97029 \qquad \text{at } x_3 = 0.03$$

Fourth Approximation:

$$y_4 = y_3 + h\,f(x_3, y_3)$$

$$\Rightarrow \quad y_4 = y_3 + h[-y_3]$$

$$\Rightarrow \quad y_4 = 0.97029 + 0.01[-0.97029] = 0.96058 \qquad \text{at } x_4 = 0.040$$

Therefore the value of y at $x = 0.04$ is 0.96058. **Answer**

(b) Given that $\frac{dy}{dx} = \log_{10}(x+y)$, $y(0)=1$, find y(0.2) using modified Euler's method.

Solution: Given differential equation is,

$$\frac{dy}{dx} = \log_{10}(x+y) = f(x, y) \qquad \dots (1)$$

with initial condition, $x_0 = 0$, $y_0 = 1$ and $x = 0.2$

Taking, $h = \frac{x - x_0}{n} = \frac{0.2 - 0}{2} = 0.1$

such that $x_1 = x_0 + h = 0 + 0.1 = 0.1$ and $x_2 = x_0 + 2h = 0 + 2(0.1) = 0.2$

1. **First Approximation:**

By Using Euler's formula

$$y_1 = y_0 + h\, f(x_0, y_0)$$

$\Rightarrow$ $y_1 = y_0 + h\left[\log_{10}(x_0 + y_0)\right] = 1 + 0.1\left[\log_{10}(0+1)\right] = 1$ at $x_1 = x_0 + h = 0.1$

Applying Euler's modified formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right] = 1.0020$$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f\left(x_1, y_1^{(1)}\right)\right] = 1.0021$ and $y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f\left(x_1, y_1^{(2)}\right)\right] = 1.0021$

Clearly, $y_1^{(3)} = y_1^{(2)} = 1.0021 = y_1$ (Improved value) at $x_1 = 0.1$

2. **Second Approximation**: Taking $y_1 = 1.0021$ and $x_1 = 0.1$ http://www.rgpvonline.com

By Using Euler's formula

$$y_2 = y_1 + h\, f(x_1, y_1)$$

$\Rightarrow$ $y_2 = y_1 + h\left[\log_{10}(x_1 + y_1)\right] = 1.0021 + 0.1\left[\log_{10}(0.1 + 1.0021)\right] = 1.0063$, at $x_2 = 0.2$

Applying Euler's modified formula,

$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2)\right] = 1.0124$, $y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f\left(x_2, y_2^{(1)}\right)\right] = 1.0125$

And $y_2^{(3)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f\left(x_2, y_2^{(2)}\right)\right] = 1.0125$

Clearly, $y_2^{(3)} = y_2^{(2)} = 1.0125 = y_2$ at $x_2 = 0.2$

Hence the required value of y at x = 0.2 is 1.0125. **Answer**

4. a) Find the Laplace transform of $f(t)=\begin{cases}1 & 0\le t<2\\ t-2 & 2\le t\end{cases}$

Solution: Given function is $f(t)=\begin{cases}1 & 0\le t<2\\ t-2 & 2\le t<\infty\end{cases}$

By the definition of Laplace Transform

$$L\{f(t)\}=\int_0^{\infty} e^{-pt}\, f(t)\, dt$$

$$\Rightarrow \qquad L\{f(t)\}=\int_0^{2} e^{-pt}\,(1)\, dt+\int_2^{\infty} e^{-pt}\,(t-2)\, dt$$

$$\Rightarrow \qquad =\left[\frac{e^{-pt}}{-p}\right]_0^2+\left[(t-2)\left\{\frac{e^{-pt}}{-p}\right\}-1\left\{\frac{e^{-pt}}{-p^2}\right\}\right]_2^{\infty}$$

$$\Rightarrow \qquad =-\frac{1}{p}\left[e^{-2p}-1\right]+\left[\{0-0\}-\left\{0+\frac{e^{-2p}}{p^2}\right\}\right]$$

$$\Rightarrow \qquad =-e^{-2p}\left[\frac{1+p}{p^2}\right]+\frac{1}{p}$$

Thus, $\boxed{L\{f(t)\}=-e^{-2p}\left[\frac{1+p}{p^2}\right]+\frac{1}{p}}$ **Answer**

(b) Write three properties of Laplace Transform.

Solution: The three properties of Laplace Transform are

(1) **Linearity of the Laplace Transform**

If a and b are anv constants and $F(t)$ and $G(t)$ be anv two function of t. then

$$L\{aF(t)+bG(t)\}=aL\{F(t)\}+bL\{G(t)\}$$

(2) **First Shifting Property:**

If $L\{F(t)\}=f(p)$, then $L\{e^{at}F(t)\}=f(p-a)$

(3) **Second Shifting property or Heaviside's Shifting Theorem:**

If $L\{F(t)\}=f(p)$ and $G(t)=\begin{cases} F(t-a), & t>a \\ 0, & t<a \end{cases}$, then $L\{G(t)\}=e^{-ap}f(p)$

5. a) What do you mean by Probability density function.

Solution: The probability density function defined on random variables.

(1) **Probability density function for discrete random variable**:

If $x_1, x_2, x_3, \ldots\ldots x_n$ are n different values of a discrete random variable X and $p(x_1)$, $p(x_2)$, $p(x_3)$ ……. $p(x_n)$ be their respective probabilities such that

(i). $p(x_i) \geq 0$, $i = 1\ 2.\ 3, \ldots\ldots n$

(ii). $\Sigma\, p(x_i) = 1$, $i = 1\ 2.\ 3, \ldots\ldots n$

then $p(x)$ is known as the **probability Mass function of the variable** X.

(2). **Probability density function for continuous random variable**:

If X is a continuous random variable and lies between two fixed value a and b, then function $f(x)$ is called pdf if $\int_a^b f(x)\,dx = 1$.

b) Explain the Binomial distribution in brief.

Solution: Hypothesis of Binomial Distribution

1. The variable should be discrete, i.e. the values of X could be 1, 2, 3, ----- etc. and never 1.2, 1.7, 3.42 etc.
2. All the trials are independent i.e. the result of one trial will not affect the result of succeeding trials.
3. The number of trials is finite and fixed say n.
4. In every trial, there are only two possible outcomes successes or failure.
5. The probability p of success is the same in every trial and q the probability of failure such that p + q = 1. This is same in all the trials.

Definition of Binomial distribution:

Let there be an event, the probability of its being success is p and the probability of its failure is q in one trial, so that $p + q = 1$ and the event be tried n times, then probability distribution of r successes with n independent trials is,

$$P(X=r) = \begin{cases} {}^nC_r\, p^r q^{n-r} \ ; & r = 0,1,2,\dots n \\ 0 \ ; & r \neq 0,1,2,\dots n \end{cases}$$

This is known as **Binomial probability distribution** and X is called the **binomial variable** and the constants n and p (or q) are called the **parameters.**

6. a) The following table is given

x :	**0**	**1**	**2**	**5**
y :	**2**	**3**	**12**	**147**

What is the form of the function?

Solution: Given:

$$x_0 = 0 \qquad x_1 = 1 \qquad x_2 = 2 \text{ and } \qquad x_3 = 5$$

and $\quad f(x_0) = 2 \qquad f(x_1) = 3 \qquad f(x_2) = 12 \qquad f(x_3) = 147$

Lagrange's interpolation formula is,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

Putting the values in above formula,

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$= -\frac{1}{5}(x^2-3x+2)(x-5) + \frac{3}{4}x(x^2-7x+10) - 2x(x^2-6x+5) + \frac{49}{20}x(x^2-3x+2)$$

$$= \frac{1}{20}(-4x^3+28x^2-68x+40+15x^3-105x^2+150x-40x^3+240x^2-200x+49x^3+147x^2+98x)$$

$$= \frac{1}{20}(20x^3+20x^2-20x+40) = x^3+x^2-x+2$$

$\Rightarrow \quad \boxed{f(x) = x^3+x^2-x+2}$ **Answer**

This is required function from the table.

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's 1/3rd rule taking $h = \frac{1}{4}$

Solution: : Given: $f(x) = \frac{1}{1+x^2}$ and $a = 0$, $b = 1$, $h = \frac{1}{4}$

$\therefore \quad n = \frac{b-a}{h} = \frac{1-0}{1/4} = 4$

Computational table:

Arguments (x)	Entry $y_x = f(x) = \frac{1}{1+x^2}$
$x_0 = a = 0$	$y_0 = \frac{1}{1+0^2} = 1$
$x_1 = x_0 + h = 0 + 1/4 = 1/4$	$y_1 = \frac{1}{1+(1/4)^2} = 0.9411$
$x_2 = x_0 + 2h = 0 + 2(1/4) = 1/2$	$y_2 = \frac{1}{1+(1/2)^2} = 0.8$
$x_3 = x_0 + 3h = 0 + 3(1/4) = 3/4$	$y_3 = \frac{1}{1+(3/4)^2} = 0.64$
$x_4 = x_0 + 4h = 0 + 4(1/4) = 1$	$y_4 = \frac{1}{1+1^2} = 0.5$

By Simpson's 1/3 rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right]$$

$$\Rightarrow \quad \int_0^1 \frac{dx}{1+x^2} = \frac{1}{12}\left[(1+0.5) + 4(0.9411 + 0.64) + 2(0.8)\right] = 0.7853$$

Answer

7. a) If 10% of the bolts produced by a machine are defective. Determine the probability that out of 10 bolts chosen at random.

(i). 1 (ii). None (iii). At most 2 bolts will be defective.

Solution: Given the number of defective bolts $n = 10$

and Probability of defective bolt $p = 10\% = 0.1$

$\therefore$ Probability of non-defective bolts $q = 1 - p = 1 - 0.1 = 0.9$

(i). The probability of 1 defective bolt $= P(X = 1)$

$$= {}^{10}C_1\, p^1 q^9 = 10(0.1)^1 (0.9)^9 = 0.3874$$ **Answer**

(ii). The probability of none defective bolt $= P(X = 0)$ http://www.rgpvonline.com

$$= {}^{10}C_0\, p^0 q^{10} = 10(0.1)^0 (0.9)^{10} = 0.3486$$ **Answer**

(iii). The probability of at most 2 defective bolts $= P(X = 0) + P(X = 1) + P(X = 2)$

$$= {}^{10}C_0\, p^0 q^{10} + {}^{10}C_1\, p^1 q^9 + {}^{10}C_1\, p^2 q^8$$

$$= 0.3486 + 0.3874 + 0.1937 = 0.9297$$ **Answer**

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 3/8th rule. **2**

Solution: : Given: $f(x) = \frac{1}{1+x^2}$ and $a = 0, \ b = 6$

Taking, $n = 6$

$\therefore \quad h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

Computational table:

Arguments (x)	Entry $y_x = f(x) = \frac{1}{1+x^2}$
$x_0 = a = 0$	$y_0 = \frac{1}{1+0^2} = 1$
$x_1 = x_0 + h = 0 + 1 = 1$	$y_1 = \frac{1}{1+1^2} = 0.5$
$x_2 = x_0 + 2h = 0 + 2 = 2$	$y_2 = \frac{1}{1+2^2} = 0.2$
$x_3 = x_0 + 3h = 0 + 3 = 3$	$y_3 = \frac{1}{1+3^2} = 0.1$
$x_4 = x_0 + 4h = 0 + 4 = 4$	$y_4 = \frac{1}{1+4^2} = 0.0588$
$x_5 = x_0 + 5h = 0 + 5 = 5$	$y_5 = \frac{1}{1+5^2} = 0.0385$
$x_6 = x_0 + 6h = 0 + 6 = 6$	$y_6 = \frac{1}{1+6^2} = 0.027$

By Simpson's 3/8th rule:

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$

$$\therefore \quad \int_0^6 \frac{dx}{1+x^2} = \frac{3(1)}{8}\left[(1+0.027) + 3(0.5+0.2+0.0588+0.0385) + 2(0.1)\right] = 1.3570$$ **Answer**

8. a) The random variable X has a Poisson distribution if P(X = 1) = 0.01487, P(X = 2) = 0.04461, then find P (X = 3).

Solution: Given:

$$P(X=1)=0.01487$$

$$\Rightarrow \quad \frac{e^{-m}m^1}{\lfloor 1} = 0.01487 \quad \Rightarrow e^{-m}\, m = 0.01487 \qquad ...(1)$$

and $\quad P(X=2)=0.04461$

$$\Rightarrow \quad \frac{e^{-m}m^2}{\lfloor 2} = 0.04461 \quad \Rightarrow e^{-m}m^2 = 0.04461 \qquad ...(2)$$

Now equation (2) divided by (1), we get

$$\therefore \quad \frac{\dfrac{e^{-m}m^2}{\lfloor 2}}{e^{-m}m} = \frac{0.04461}{0.01487}$$

$$\Rightarrow \quad \frac{m}{2} = 3 \quad \text{i.e. } m = 6$$

Now, $P(X=3) = \dfrac{e^{-m}m^3}{\lfloor 3} = \dfrac{e^{-6}(6)^3}{6} = 0.08923$ **Answer**

b) Find the Fourier transform of $F(x) = \begin{cases} 1 \; ; \; |x| < a \\ 0 \; ; \; |x| > a \end{cases}$

Solution: Given the function $F(x) = \begin{cases} 1 \; ; \; -a < x < a \\ 0 \; ; \; |x| > a \end{cases}$ ... **(1)**

The Fourier transform of a function $F(x)$ is given by

$$f(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x)\, e^{ipx}\, dx$$

$\Rightarrow$
$$f(p) = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} 1 . e^{ipx}\, dx = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{ipx}}{ip} \right]_{-a}^{a}$$

$\Rightarrow$
$$= \frac{1}{\sqrt{2\pi}} \left(\frac{2}{p} \right) \left[\frac{e^{ipa} - e^{-ipa}}{2i} \right] = \sqrt{\frac{2}{\pi}} \frac{\sin ap}{p}$$

Thus, $\boxed{f(p) = \sqrt{\frac{2}{\pi}} \frac{\sin ap}{p}}$ **Answer**

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