RGPV SOLUTION BE-3001 MATHEMATICS-III DEC 2017

Branch: EC/EI/EE

1. (a) Find Fourier series for $f(x) = e^x$ in the interval $(-\pi, \pi)$

Solution : Given : $f(x) = e^x, \; -\pi < x < \pi$ …………..(1)

Here, $2L = \pi - (-x)$ i.e. $2L = 2\pi \Rightarrow L = \pi$

Suppose the Fourier series of $f(x)$ with period $2L$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$\Rightarrow \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$ [Since $L = \pi$] …………(2)

Now, $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x dx$

$$= \frac{1}{\pi}\left[e^x\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left[e^{\pi} - e^{-\pi}\right]$$

$$\Rightarrow \quad a_0 = \frac{2\sin h\pi}{\pi}$$

and $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \cos nx \, dx$

$$= \frac{1}{\pi}\left[\frac{e^x}{1^2 + n^2}(\cos nx + \sin nx)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{(n^2+1)\pi}\left[e^x(\cos nx + \sin nx)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{(n^2+1)\pi}\left[\left\{e^{\pi}(\cos n\pi + \sin n\pi\right\} - \left\{e^{-\pi}(\cos n\pi - \sin n\pi\right\}\right]$$

$$= \frac{1}{(n^2+1)\pi}\left[\left\{e^{\pi}(-1)^n + 0\right\} - \left\{e^{-\pi}((-1)^n - 0)\right\}\right]$$

$$= \frac{(-1)^n}{(n^2+1)\pi}\left[e^{\pi} - e^{-\pi}\right] = \frac{2(-1)^n \sinh \pi}{(n^2+1)\pi}$$

$$\therefore \quad \boxed{a_n = \frac{2(-1)^n \sinh \pi}{(n^2+1)\pi}}$$

Now, $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} e^x \sin nx \, dx$

$$= \frac{1}{\pi}\left[\frac{e^x}{1^2+n^2}\left(\sin nx - \cos nx\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\left(n^2+1\right)\pi}\left[e^x\left(\sin nx - \cos nx\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\left(n^2+1\right)\pi}\left[\left\{e^{\pi}(\sin n\pi - \cos n\pi\right\} - \left\{e^{-\pi}(-\sin n\pi - \cos n\pi\right\}\right]$$

$$= \frac{1}{\left(n^2+1\right)\pi}\left[\left\{e^{\pi}\left(0-(-1)^n\right)\right\} - \left\{e^{-\pi}\left(0-(-1)^n\right)\right\}\right]$$

$$= \frac{-\left(-1\right)^n}{\left(n^2+1\right)\pi}\left[e^{\pi} - e^{-\pi}\right]$$

$\therefore$ $$\boxed{b_n = \frac{2\left(-1\right)^{n+1}\sinh\pi}{\left(n^2+1\right)\pi}}$$

Putting in equation (1), we get

$$\boxed{f(x) = \frac{\sinh\pi}{\pi} + \frac{2\sinh\pi}{\pi}\sum_{n=1}^{\infty}\frac{\left(-1\right)^n\cos nx}{n^2+1} - \frac{2\sinh x}{\pi}\sum_{n=1}^{\infty}\frac{\left(-1\right)^n\sin nx}{n^2+1}}$$ **Answer**

(b) Express *f(x)=x* as a half range sine series in 0 < *x* < 2

Solution : Given : $f(x) = x, \quad 0<x<2$ ………(1)

Here, $L = 2$

Suppose the Half range sine series of $f(x)$ is,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

$$\Rightarrow \quad f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right)$$ [Since $L = \pi$] ……(2)

Now, $b_n = \frac{2}{2}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)dx = \int_0^2 x\sin\left(\frac{n\pi x}{2}\right)dx$

$$\Rightarrow \quad \left[x\left(-\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\right) - 1\left(-\frac{4}{n^2\pi^2}\right)\sin\left(\frac{n\pi x}{2}\right)\right]_0^2$$

$$\Rightarrow \quad = \left\{-\frac{4}{n\pi}\left(-1\right)^n - 0\right\} - \{0-0\}$$

$$\Rightarrow \quad b_n = -\frac{4}{n\pi}(-1)^n$$

Putting in equation (1), we get

$$\boxed{f(x) = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi x}{2}\right)}$$ **Answer**

2. (a) Find Fourier cosine transform of $f(x) = e^{-x}$

Solution : Given the function : $F(x) = e^{-ax}$

The Fourier cosine transform of $F(x)$ is given by,

$$f_c(p) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F(x)\cos px\,dx$$

$$\Rightarrow \quad f_c(p) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-x}\cos px\,dx = \sqrt{\frac{2}{\pi}}\left[\frac{e^{-x}}{(-1)^2 + p^2}\{-\cos px + p\sin px\}\right]_0^{\infty}$$

$$f_c(p) = \frac{-1}{p^2+1}\sqrt{\frac{2}{\pi}}\left[e^{-x}\{\cos px - p\sin px\}\right]_0^{\infty} = \frac{1}{p^2+1}\sqrt{\frac{2}{\pi}}\left[\{0 - 1(1 + p.0)\}\right]$$

Thus $$\boxed{f_c(p) = \sqrt{\frac{2}{\pi}}\left[\frac{1}{p^2+1}\right]}$$ **Answer**

(b) Find a Fourier series of represent $f(x) = x$ **from** $(-\pi, \pi)$**.**

Solution : Given : $f(x) = x, \ -\pi < x < \pi$ …….(1)

Here, $2L = \pi - (-\pi)$ i.e. $2L = 2\pi \Rightarrow L = \pi$

Suppose the Fourier series of $f(x)$ with period $2L$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n\sin\left(\frac{n\pi x}{L}\right)$$

$$\Rightarrow \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx \qquad [\text{Since } L = \pi\ ] \ \ldots\ldots(2)$$

Now, $$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x dx = 0 \qquad [\text{Since } x = \text{odd}]$$

and $$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx dx \qquad [\ x\cos nx = \text{odd})$$

$$\Rightarrow \quad = \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$\Rightarrow \quad = \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - 1\left(\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$\Rightarrow \quad a_n = \frac{2}{\pi}\left[\left\{0 + \frac{(-1)^n}{n^2}\right\} - \left\{0 + \frac{1}{n^2}\right\}\right] = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

and $\quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx$

$\Rightarrow \quad = 0 \qquad [\,x\sin nx = \text{odd})$

Putting in equation (1), we get

$$f(x) = 0 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left[(-1)^n - 1\right]}{n^2}\cos nx + 0$$

$\Rightarrow \quad \boxed{f(x) = -\frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...\right]}$ **Answer**

3. (a) Find Laplace transform of the following functions :

(i). $\frac{\sin t}{t}$ **(ii).** $t\,e^{at}\sin t$

Solution : (i). Let $F(t) = \sin t$

$\therefore \quad L\{F(t)\} = L\{\sin t\} = \frac{1}{p^2+1} = f(p)$

By Laplace transform of division of t, we have

$$L\left\{\frac{F(t)}{t}\right\} = \int_p^{\infty} f(p)dp \qquad \text{............(1)}$$

$\therefore \quad L\left\{\frac{\sin t}{t}\right\} = \int_p^{\infty}\frac{1}{p^2+1}dp = \left[\tan^{-1} p\right]_p^{\infty}$

$\Rightarrow \quad = \tan^{-1}(\infty) - \tan^{-1}(p) = \frac{\pi}{2} - \tan^{-1}(p) = \cot^{-1}(p)$ **Answer**

(ii). $\quad L\{\sin t\} = \frac{1}{p^2+1} = f(p)$

By Multiplication property, we have

$$L\{t\sin t\} = (-1)\frac{d}{dp}f(p)$$

$\Rightarrow \quad L\{t\sin t\} = -\frac{d}{dp}\left(\frac{1}{p^2+1}\right) = \frac{2p}{(p^2+1)^2} = f_1(p)$

By First Shifting property

$$L\{e^{at}(t\sin t)\} = f_1(p-a)$$

$\Rightarrow \quad L\{e^{at}(t\sin t)\} = \frac{2(p-a)}{\left[(p-a)^2+1\right]^2}$ **Answer**

(b) Using convolution theorem to find inverse Laplace transforms of $\frac{s}{(s-a)(s-b)}$

Solution : Given $\frac{s}{(s-a)(s-b)} = \frac{(s-a)+a}{(s-a)(s-b)} = \frac{1}{s-b} + \frac{a}{(s-a)(s-b)}$

Now $L^{-1}\left\{\frac{s}{(s-a)(s-b)}\right\} = L^{-1}\left\{\frac{1}{s-b}\right\} + aL^{-1}\left\{\frac{1}{(s-a)(s-b)}\right\}$

$$L^{-1}\left\{\frac{s}{(s-a)(s-b)}\right\} = e^{bt} + aL^{-1}\left\{\frac{1}{(s-a)(s-b)}\right\} \qquad \text{……….(1)}$$

Suppose $f(s) = \frac{1}{s-a}$ and $g(s) = \frac{1}{s-b}$

$\therefore$ $L^{-1}\{f(s)\} = L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at} = F(t)$

And $L^{-1}\{g(s)\} = L^{-1}\left\{\frac{1}{s-b}\right\} = e^{bt} = G(t)$

By Convolution theorem of Inverse Laplace transform, we have

$$L^{-1}\{f(s)g(s)\} = \int_0^t F(x)G(t-x)dx$$

$\therefore$ $L^{-1}\left\{\frac{1}{(s-a)(s-b)}\right\} = \int_0^t \left[e^{ax}\right]\left[e^{b(t-x)}\right]dx$

$$= e^{bt}\int_0^t e^{(a-b)x}dx$$

$$= e^{bt}\left[\frac{e^{(a-b)x}}{(a-b)}\right]_0^t$$

$$= -e^{bt}\left[e^{(a-b)t} - 1\right]$$

$$= \frac{1}{a-b}\left[e^{at} - e^{bt}\right]$$

Putting in equation (1), we get

$$\boxed{L^{-1}\left[\frac{s}{(s-a)(s-b)} = e^{bt} + \frac{a}{a-b}\left[e^{at} - e^{bt}\right]\right]}$$ **Answer**

4. (a) Test the analyticity of the function $w = e^x$

Solution : Suppose $f(z) = e^z = e^{x+iy} = e^x e^{iy}$

$\Rightarrow$ $f(z) = e^x(\cos y + i\sin y)$

$\Rightarrow \qquad u + iv = e^x \cos y + ie^x \sin y$

Equating on both sides, we get

$$u = e^x \cos y \text{ and } v = e^x \sin y$$

Partially differentiating with respect to, x and y, we get

$\frac{\partial u}{\partial x} = e^x \cos y$	$\frac{\partial v}{\partial x} = e^x \sin y$
$\frac{\partial u}{\partial y} = -e^x \sin y$	$\frac{\partial v}{\partial y} = e^x \cos y$

Clearly, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Therefore, C-R equation is satisfied, then given function is analytic everywhere

(b) Using Cauchy's residue theorem, evaluate the real integral $\int_C \frac{e^{2z}}{z(z-1)} dz,$

Where c is the circle $|z| = \frac{1}{2}$

Solution : Given, $I = \int_C \frac{e^{2z}}{z(z-1)} dz$

The pole of integrand is given by,

$$z(z-1) = 0 \Rightarrow z = 0,1$$

Now, $\quad z = 0 \Rightarrow |z| = 0 < \frac{1}{2}$ [Lies within C]

and $\quad$ z = 1 $\Rightarrow |z = 1 > \frac{1}{2}$ [Outside the region of C]

By Cauchy integral formula,

$$\int_C \frac{e^{2z}}{z(z-1)} dz = \int_{C_1} \frac{\frac{e^{2z}}{z}}{z-1} dz$$

$$\Rightarrow \qquad = 2\pi i \left[\frac{e^{2z}}{z} \right]_{z=1}$$

$$\Rightarrow \qquad = 2\pi i \left[\frac{2^{2(1)}}{1} \right]$$

$$\Rightarrow \qquad = 2\pi i \, e^2$$

Thus, $\boxed{\int_C \frac{e^{2z}}{z(z-1)}dz = 2\pi i\, e^2}$ **Answer**

5. (a) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and find its harmonic conjugate.

Solution : Given : $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

Partially differentiate w.r.t. x and y respectively

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 6x$ and $\frac{\partial^2 u}{\partial x^2} = 6x + 6$ …………(1)

$\frac{\partial u}{\partial y} = -6xy - 6y$ and $\frac{\partial^2 u}{\partial y^2} = -6x - 6$ ………….(2)

Adding (1) and (2), we get

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$\therefore$ u is harmonic function.

To Find Conjugate function v

Now, $dv = \left(\frac{\partial v}{\partial x}\right)dx + \left(\frac{\partial v}{\partial y}\right)dy$

$\Rightarrow \quad dv = \left(-\frac{\partial u}{\partial y}\right)dx + \left(\frac{\partial u}{\partial x}\right)dy$ [by Cauchy-Riemann Equation]

$\Rightarrow \quad = (6xy + 6y)dx + (3x^2 - 3y^2 + 6x)dy$

Integrating both sides, we get

$$v = \int_{\text{y constant}} (6xy + 6y)dx + \int_{\text{Independent of x}} (-3y^2)dy + c$$

$\boxed{v = 3x^2 y + 6xy - y^3 + c}$ **Answer**

(b) Evaluate $\int_C z^2 dz$, where c is the straight line joining the points (0, 0) and (2, 2).

Solution : The equation of straight line joining the points (0, 0) and (2, 2) is

$$y - 0 = \frac{2-0}{2-0}(x - 0) \Rightarrow y = x \text{ and } dy = dx$$

Since $z = x + iy = x + xi$ so that $dz = (1+i)dx$

and $z^2 = (x+iy)^2 = (x+ix)^2 = x^2(1+i)^2$

Now, $I = \int_C z^2 dz = \int_0^1 \left[x^2(1+i)^2\right](1+i)dx$

$$= (1+i)^3 \int_0^1 x^2 dx = (1-i+3i-3)\left[\frac{x^3}{3}\right]_0^1$$

$$= (-2+2i)\left[\frac{1}{3}-0\right] = -\frac{2}{3}+\frac{2}{3}i$$

$\therefore$ $\boxed{\int_C z^2 dx = -\frac{2}{3}+\frac{2}{3}i}$ **Answer**

6. (a) Evaluate the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q has coordinates (5, 0, 4).

Solution : Given the scalar function is $\phi = x^2 - y^2 + 2z^2$

Now, $grad\,\phi = \left(\hat{i}\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)(x^2 - y^2 + 2z^2)$

$$= \hat{i}\frac{\partial}{\partial x}(x^2 - y^2 + 2z^2) + j\frac{\partial}{\partial y}(x^2 - y^2 + 2z^2) + k\frac{\partial}{\partial z}(x^2 - y^2 + 2z^2)$$

$$= 2x\hat{i} - 2yj + 4zk$$

$\therefore$ $grad\,\phi = 2\hat{i} - 4j + 12k$ at P(1, 2, 3)

Suppose $\vec{a} = \overline{PQ} = \overline{OQ} - \overline{OP} = \left(5\hat{i} - 0j + 4k\right) - \left(\hat{i} + 2j + 3k\right) = 4\hat{i} - 2j + k$

Let a be unit vector along the direction of $\overline{PQ}$, then

$$a = \frac{4\hat{i} - 2j + k}{\sqrt{4^2 + (-2)^2 + 1^2}} = \frac{4\hat{i} - 2j + k}{\sqrt{21}}$$

The D.D. of scalar function ϕ at the point P(1, 2, 3) in the direction of $\vec{a}$ is

D.D. = a. grad ϕ

$\Rightarrow$ $= \left(\frac{4\hat{i} - 2j + k}{\sqrt{21}}\right) . \left(2\hat{i} - 4j + 12k\right) = \frac{1}{\sqrt{21}}(8+8+12) = \frac{28}{\sqrt{21}}$

$\therefore$ $\boxed{D.D. = \frac{28}{\sqrt{21}}}$ **Answer**

(b) Use Stoke's theorem to evaluate $\int_C [([2x-y])dx - yz^2 dy - y^2 z dz]$, where c is the circle $x^2 + y^2 = 1$, corresponding to the surface of sphere of unit radius.

Solution : Given $I = \int_C [(2x-y)dx - yz^2\, dy - y^2 z\, dz]$

$\therefore \quad F = (2x-y)\hat{i} - yz^2\,\hat{j} - y^2 z\,\hat{k}$

By Stock's theorem we have

$$\int_C F\, dr = \iint_S CurlF\, n\, ds$$

Now, $CurlF = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2 z \end{vmatrix}$

$$= \hat{i}(-2yz + 2yz) - \hat{j}(0-0) + \hat{k}(0-1)$$

$$= \hat{k}$$

Since the surface on the dy-plane, then $n = \hat{k}$

And $\quad CurlF\, n = \left(\hat{k}\right)\left(\hat{k}\right) = 1$

The projection on XY-plane then we have

$$\iint_S CurlF\, n\, ds = \iint_{S_1} \frac{Curl\, F\, n}{|n.\hat{k}\,|} dx\, dy$$

$$= \iint_{S_1} 1\, dx\, dy = \iint_{S_1} dx\, dy$$

= Area of circle in xy plane

$$= \pi(1)^2 = \pi$$

Hence, $\boxed{\int_C F\, dr = \iint_S CurlF\, n\, ds = \pi}$ **Answer**

7. (a) **A vector field is given by** $A=\left(x^2+xy^2\right)\hat{i}+\left(y^2+x^2y\right)j$. Show that the vector field is irrotational.

Solution : Given $A=\left(x^2+xy^2\right)\hat{i}+\left(y^2+x^2y\right)j$

Now, $$Curl\,A=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ x^2+xy^2 & y^2+x^2y & 0\end{vmatrix}$$

$\Rightarrow$ $$=\hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(2xy-2xy)$$

$\Rightarrow$ $$=0\,\hat{i}+0\,\hat{j}+0\,\hat{k}=0$$

$\therefore$ A is irrotational vector. **Answer**

(b) **Define the divergence of a vector field and show that the vector**

$A=(x+3y)\hat{i}+(y-3z)j+(x-2z)k$ **is solenoidal.**

Solution : In vector calculus, the divergence is an operator that measures the magnitude of a vector field's source or sink at a given point; the divergence of a vector field is a (signed) scalar. For example, for a vector field that denotes the velocity of air expanding as it is heated, the divergence of the velocity field would have a positive value because the air expands. If the air cools and contracts, the divergence is negative.

Now $$div\,\overline{A}=\nabla\overline{A}$$

$\Rightarrow$ $$=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\left(xy^2\,\hat{i}+2x^2yz\,\hat{j}-3yz^2\,\hat{k}\right)$$

$\Rightarrow$ $$=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\left[(x+3y)\hat{i}+(y-3z)j+(x-2z)k\right]$$

$\Rightarrow$ $$=\frac{\partial}{\partial x}(x+3y)+\frac{\partial}{\partial y}(y-3z)+\frac{\partial}{\partial z}(x-2z)=1+1-2=0$$

$\therefore$ $\overline{A}$ is solenoidal. **Hence Proved**

8. (a) Using Laplace transform, solve $\frac{d^2y}{dt^2} - 4y = 24\cos 2t,$ **given that** $y(0) = 3, y'(0) = 4$

Solution : Given the dirrerential equation is,

$$y''(t) - 4y(t) = 24\cos 2t \qquad \text{..........(1)}$$

With initial condition are: $y(0) = 3 \, and \, y'(0) = 4$

Taking Laplace transform of (1) on both sides, we get

$$L\{y''(t)\} - 4L\{y(t)\} = 24L\{\cos 2t\}$$

$$\Rightarrow \quad [p^2 y(p) - py(0) - y'(0)] - 4y(p) = \frac{24p}{p^2+4}$$

Putting the initial values, $y(0) = 3 \, and \, y'(0) = 4, we \, ge$

$$\therefore \quad [p^2 y(p) - 3p - 4] - 4y(p) = \frac{24p}{p^2+4}$$

$$\Rightarrow \quad (p^2 - 4)y(p) = \frac{24p}{p^2+4} + 3p + 4$$

$$\Rightarrow \quad y(p) = \frac{24p}{(p^2-4)(p^2+4)} + \frac{3p}{p^2-4} + \frac{4}{p^2-4}$$

$$\Rightarrow \quad y(p) = \frac{24}{8} p \left[\frac{1}{p^2-4} - \frac{1}{p^2+4}\right] + \frac{3p}{p^2-4} + \frac{4}{p^2-4}$$

$$\Rightarrow \quad L\{y(t)\} = \frac{3p}{p^2-4} - \frac{3p}{p^2+4} + \frac{3p}{p^2-4} + \frac{4}{p^2-4} = \frac{6p}{p^2-4} - \frac{3p}{p^2+4} - \frac{4}{p^2-4}$$

$$\Rightarrow \quad y(t) = L^{-1}\left\{\frac{6p}{p^2-4}\right\} - L^{-1}\left\{\frac{3p}{p^2+4}\right\} - L^{-1}\left\{\frac{4}{p^2-4}\right\}$$

$$\Rightarrow \quad y(t) = 6\cosh 2t - 3\cos 2t - 2\sinh 2t$$

Thus, $\boxed{y(t) = 6\cosh 2t - 3\cos 2t - 2\sinh 2t}$ **Answer**

(b) Find the following:

(i). $L\{e^{-3t}\cos 4t\}$ (ii). $L^{-1}\left\{\frac{3s+5}{s^2-2s-3}\right\}$

Solution : (i). $L\{\cos 4t\} = \frac{p}{p^2+16} = f(p)$

$$L\{e^{-3t}\cos 4t\} = f(p+4)$$

$\Rightarrow \qquad = \dfrac{p+3}{(p+3)^2+16} = \dfrac{p+3}{p^2+6p+25}$

$\therefore \qquad \boxed{L\{e^{-3t}\cos 4t\} = \dfrac{p+3}{p^2+6p+25}}$ **Answer**

(ii). $L^{-1}\left\{\dfrac{3s+5}{s^2-2s-3}\right\} = L^{-1}\left\{\dfrac{3s+5}{(s-3)(s+1)}\right\}$

$= L^{-1}\left\{\dfrac{3(3)+5}{\boxed{(s-3)}\,(3+1)} + \dfrac{3(-1)+5}{(-1-3)\,\boxed{(s+1)}}\right\}$

$= \dfrac{14}{4}L^{-1}\left\{\dfrac{1}{s-3}\right\} - \dfrac{2}{4}L^{-1}\left\{\dfrac{1}{s+1}\right\}$

$= \dfrac{7}{2}e^{3t} - \dfrac{1}{2}e^{-t}$

$\therefore \qquad \boxed{L^{-1}\left\{\dfrac{3s+5}{s^2-2s-3}\right\} = \dfrac{7}{2}e^{3t} - \dfrac{1}{2}e^{-t}}$ **Answer**

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