OR

Find for what value of λ and μ, the equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

have

i) No solution

ii) A unique solution

iii) Infinite many solutions

Unit - V

5. a) Define each of the following and give examples :

i) Graph ii) Digraph

iii) Pseudo graph iv) Order of a graph

b) Construct the truth table for the proposition :

i) $\sim p \wedge q$ ii) $p \wedge (p \vee q)$

c) For any Boolean algebra (B,+, $\cdot$, $'$), prove that

i) Additive identity is unique

ii) Multiplicative identity is unique

iii) For each $a \in B$, complement a' is unique.

d) Prove that a simple graph with n vertices and k components can have at most $\frac{1}{2}(n-k)(n-k+1)$ edges.

OR

Prove that every non-trivial tree has atleast 2-vertices of degree 1.

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Roll No ....

BE - 102

B.E. I & II Semester

Examination, June 2015

Engineering Mathematics-I

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each questions are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

Unit - I

1. a) Apply Maclaurin's theorem to prove that

$$\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \ldots\ldots\ldots.$$

b) Show that $\log(x+h) = \log h + \frac{x}{h} - \frac{x^2}{2h^2} + \frac{x^3}{3h^3} + \ldots\ldots\ldots$.

c) Discuss the maximum and minimum of $x^3 + y^3 - 3xy$.

d) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u.\sin u$$

OR

If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters of an ellipse, prove that

$$\left\{(\rho_1)^{2/3}+(\rho_2)^{2/3}\right\}(ab)^{2/3}=a^2+b^2.$$

Unit - II

2. a) Prove that $\int_0^1 \frac{\log(1+x)}{1+x^2}dx=\frac{\pi}{8}\log 2$.

b) Evaluate $\int_a^b \cos x\,dx$ as limit of sums.

c) Show that $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1}dy=\frac{\overline{|(p)}}{q^p}$, where $p,q>0$.

d) Change the order of integration of $\int_0^a\int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dxdy$ and hence evaluate it.

OR

Express the area between the curves $x^2+y^2=a^2$, and $x+y=a$ as double integral and evaluate it.

Unit - III

3. a) Solve $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

b) Solve $y^2\log y=xyp+p^2$.

c) Solve $(D^3+3D^2+2D)y=x^2$.

d) Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\log x$.

OR

Solve by method of variation of parameters

$$\frac{d^2y}{dx^2}+4y=4\tan 2x.$$

Unit - IV

4. a) Find the rank and nullity of the following matrix

$$A=\begin{bmatrix}1 & a & b & 0\\ 0 & c & d & 1\\ 1 & a & b & 0\\ 0 & c & d & 1\end{bmatrix}.$$

b) Show that the following system of equation is inconsistent:

$x-2y+z-w=-1$

$3x-2z+3w=-4$

$5x-4y+w=-3$

c) Find Eigen values and Eigen vectors of: $A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$.

d) Verify Cayley-Hamilton theorem for the matrix :

$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$. Hence find A^{-1}.