

BE-102**B.E. I & II Semester**

Examination, June 2016

Engineering Mathematics - I**Time : Three Hours****Maximum Marks : 70**

- Note:** i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.
 ii) All parts of each question are to be attempted at one place.
 iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max. 400 words) carry 7 marks.
 iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Define radius of curvature and centre of curvature.
 b) If $u = x^3 + y^3 + 3xy$, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.
 c) Find the first three terms in the expansion of $\log(1 + \tan x)$ by Maclaurin's theorem.
 d) Discuss the maxima or minima value of $u = f(x) = x^3 - y^2 - 7x^2 + 4y + 15x - 13$.

OR

If $u = \sec^{-1} \left(\frac{x^2 + y^2}{x - y} \right)$. Find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

2. a) Define Gamma and Beta function.

b) Evaluate $\int_1^2 x dx$, as the limit of a sum.

c) Evaluate the integral $\iiint xyz dz dy dx$,

Over the volume enclosed by three co-ordinates planes and the plane $x + y + z = 1$.

d) Prove that $\Gamma(m) \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$.

OR

Change the order of integration

$$\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy dx.$$

3. a) Define linear and non-linear ordinary differential equation.

b) Solve $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$.

c) Solve $y = 2px + y^2 p^3$.

d) Solve $\frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = e^{-2x} \sin 2x$.

OR

Solve $\frac{dx}{dt} + y = \sin t$

and $\frac{dy}{dt} + x = \cos t$.

4. a) Define rank of a matrix.

b) Find the rank of a matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

c) Examine for consistency, the following equations:

$$5x + 3y + 14z = 4,$$

$$y + 2z = 1,$$

$$x - y + 2z = 0,$$

$$2x + y + 6z = 2$$

d) Find Eigen values and Eigen vectors of the following

matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

OR

Verify Cayley-Hamilton theorem for the following

matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also find A^{-1} .

5. a) Define simple graph and tree.

b) Explain elementary concept of fuzzy logic.

c) For a Boolean algebra B, prove that

$$(x \cdot y' + y \cdot z) \cdot (x \cdot z + y \cdot z') = x \cdot z$$

d) Draw the switching circuit of the following Boolean function and simplified it.

$$f(x, y, z) = x \cdot y \cdot z + x \cdot y' \cdot z + x' \cdot y' \cdot z$$

Or

For Boolean algebra B, prove that

i) $(a + b)' = a' \cdot b'$, $\forall a, b \in B$

ii) $(a \cdot b)' = a' + b'$, $\forall a, b \in B$
