

5. a) Find the directional derivative of  $\phi = xy + yz + zx$  in the direction of  $2i + j + k$  at the point  $(1, 1, 2)$ . Also find the maximum value of the directional derivative at the point.
- b) Show that the vector  $F = \frac{\bar{r}}{r^3}$  is irrotational. Find the scalar potential.

OR

- a) Evaluate  $\iint_s A \cdot \hat{n} ds$ , where  $A = (x + y^2) i - 2xj + 2yzk$  and  $s$  is the surface of the plane  $2x + y + 2z = 6$  in the first octant.
- b) Using Stoke's theorem, evaluate

$$\int_C [(x + y)dx + (2x - z)dy + (y + z)dz]$$

Where  $C$  is the boundary of the triangle with vertices  $(2, 0, 0)$ ,  $(0, 3, 0)$  and  $(0, 0, 6)$

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Roll No .....

### BE-301

### B.E. III Semester

Examination, December 2016

### Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70

- Note:** i) Attempt all questions. Each question has an internal choice.
- ii) All questions carry equal marks.
- iii) All parts of each question are to be attempted at one place.

1. a) Expand in Fourier series  $f(x) = x + x^2, -\pi < x < \pi$
- b) Obtain half-range Fourier cosine series for  $f(x) = x$  in  $0 < x < 2$

OR

- a) Find the Fourier series for the even function

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \dots = \frac{\pi^2}{8}$$

b) Find the Fourier sine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

2. a) Find the L.T. of

$$i) f(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 1, & t > 0 \end{cases}$$

$$ii) f(t) = t^2 \sin 2t$$

b) Use convolution theorem, to find

$$L^{-1} \left\{ \frac{1}{s^3(s^2+1)} \right\}$$

OR

a) Using L.T. techniques, solve the following initial value problem:

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4e^{2t}, y(0) = -3, y'(0) = 5$$

b) Find the L.T. of the triangular wave function of period 2k given by:

$$f(t) = \begin{cases} t, & 0 < t < k \\ 2k-t, & k < t < 2k \end{cases}$$

3. a) Solve  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$ , given that  $y = x + \frac{1}{x}$  is one integral.

b) Find the general solution of  $\frac{d^2y}{dx^2} + (x-3)\frac{dy}{dx} + y = 0$  in series.

OR

a) Solve

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \cos \log(1+x)$$

b) Solve by method of variation of parameters:

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

4. a) Solve  $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

b) Solve  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$

OR

a) Solve the following partial differential equation by Charpit's method:

$$pxy + pq + qy = yz$$

b) A tightly stretched string of length L is fixed at both ends. Find the displacement  $u(x, t)$ , if the string is given an initial displacement  $f(x)$  and an initial velocity  $g(x)$ .