Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. ....

CS/IT–302(GS)

B. E. (Third Semester) EXAMINATION, Dec., 2011

(Grading System)

(Common for CS & IT Engg.)

DISCRETE STRUCTURE

Time : Three Hours

Maximum Marks : 70

Minimum Pass Marks : 22 (Grade – D)

Note : The question paper is divided into five Unit. Each Unit carries an internal choice. Attempt *one* question from each Unit. All questions carry equal marks.

Unit – I

1. (a) Out of 120 students surveyed, it was found that 20 students have studied French, 50 students have studied English, 70 students have studied Hindi, 5 have studied English and French, 20 have studied English and Hindi, 10 have studied Hindi and French. Only 3 students have studied all the three languages. Find how many students have studied :

   (i) Hindi alone

   (ii) French alone

   (iii) English, but not Hindi

   (iv) Hindi, but not French

   (b) Use mathematical induction to prove that $2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24 for all $n > 0$.

2. (a) Let R be the relation defined on the integers by $a\text{R}b$ if $a - b$ is even. Show that R is an equivalence relation and determine the equivalence classes.

   (b) Show that the mapping $f : Z^+ \rightarrow Z^+$ defined by $f(x) = x^2, \forall \ x \in Z^+$ where Z^+ is a set of positive integers, is one to one and onto.

Unit – II

3. (a) Let $(A, *)$ be a monoid such that for every x in A, $x * x = e$ where e is the identity element. Show that $(A, *)$ is an abelian group.

   (b) Let $(A, +, \cdot)$ be a ring such that $a \cdot a = a$ for all a in A :

   (i) Show that $a + a = 0 \ \forall \ a \in A$, where 0 is the additive identity.

   (ii) Show that the operation ' $\cdot$ ' is commutative.

4. (a) Let $(H, \cdot)$ be subgroup of a group $(G, \cdot)$. Let $N = \{x \mid x \in G, x H x^{-1} = H\}$. Show that $(N, \cdot)$ is a subgroup of $(G, \cdot)$.

   (b) Let S be the set of real numbers of the form $a + b\sqrt{3}$ where a and b are rational numbers. Show that S is a field with respect to addition and multiplication.

Unit – III

5. (a) Determine whether the following proposition is contradiction or a tautology, where p and q are propositions :

$$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$$

   (b) Show that the following Language is not a finite state language :

$$L = \{1^i 0^j 1^{i+j} \mid i \geq 1, j \geq 1\}$$

6. (a) Obtain the principle conjunctive normal form of :

$$(\sim p \Rightarrow r) \wedge (q \Leftrightarrow p)$$

(b) For the Finite state machine shown below :

(i) List all 0, equivalent states.

(ii) Find all equivalent states and obtains an equivalent finite state machine with the smallest number of states :

State	Input		Output
	0	1	
⇒ A	F	B	0
B	D	C	0
C	G	B	0
D	E	A	1
E	D	A	0
F	A	G	1
G	C	H	1
H	A	H	1

Unit – IV

7. (a) Define the following :

(i) Regular graph

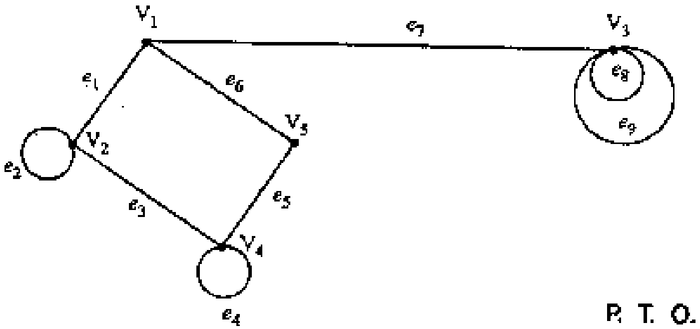
(ii) Homeomorphism graph

(iii) Eulerian graph

(iv) Hamiltonian graph

(v) Eccentricity and centre of a tree

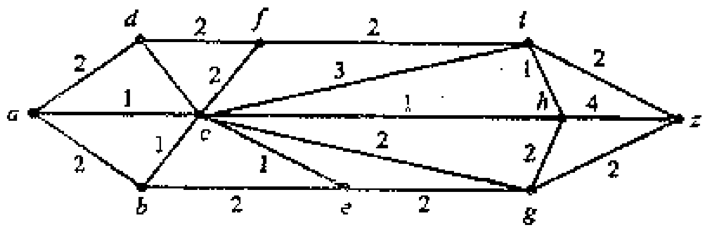
(b) Write down the adjacency and incidence matrix of the graph given below :



8. (a) Prove that a graph G with n vertices always has a Hamiltonian path if the sum of the degrees of every pair of vertices V_i, V_j in G satisfies the condition :

$$d(V_i) + d(V_j) \geq (n - 1)$$

(b) Find the shortest path for the following graph.



Unit – V

9. (a) Let a, b, c be the elements in a lattice $(A, \leq)$. Show that if $a \leq b$ then :

$$a \vee (b \wedge c) \leq b \wedge (a \vee c)$$

(b) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 12$ satisfy the recurrence relation $a_r + c_1 a_{r-1} + c_2 a_{r-2} = 0$, determine a_r.

10. (a) Determine the discrete numeric function carresponds to each of the following generating functions :

(i) $A(z) = \dfrac{7z^2}{(1 - 2z)(1 + 3z)}$

(ii) $A(z) = \dfrac{(1 + z)^2}{(1 - z)^4}$

(b) Find the total solution of the recurrence relation :

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r, \; r \geq 2$$

with boundary conditions $a_0 = 1$ and $a_1 = 1$.