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Roll No ....

BE-3001 (EX/EI/EE) (CBGS)

B.E., III Semester

Examination, December 2017

Choice Based Grading System (CBGS)

Mathematics - III

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions out of eight.

ii) All questions carry equal marks.

1. a) Find Fourier series of the function $f(x) = e^x$ in the interval $(-\pi, \pi)$.

   b) Express $f(x) = x$ as a half range sine series in $(0 < x < 2)$.

2. a) Find Fourier cosine transform of e^{-x}.

   b) Find a Fourier series of represent $f(x) = x$ from $(-\pi, \pi)$.

3. a) Find Laplace transform of the following functions:

   i) $\frac{\sin t}{t}$ and   ii) $te^{at}\sin t$

   b) Using convolution theorem to find inverse Laplace transforms of $\frac{s}{(s-a)(s-b)}$.

4. a) Test the analyticity of the function $w = e^z$.

   b) Using Cauchy's residue theorem, evaluate the real integral

   $\int_c \frac{e^{2z}}{z(z-1)} dz$, where c is the circle $|z| = \frac{1}{2}$.

5. a) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and find its harmonic conjugate.

   b) Evaluate $\int_C (z^2) dz$, where C is the straight line joining the points (0, 0) and (2, 2).

6. a) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4).

   b) Use Stoke's theorem to evaluate $\int_c \left[(2x-y)dx - yz^2dy - y^2zdz\right]$, where c is the circle $x^2 + y^2 = 1$, corresponding to the surface of spheres of unit radius.

7. a) A vector field is given by $\vec{A} = \left(x^2 + xy^2\right)\hat{i} + \left(y^2 + x^2y\right)\hat{j}$. Show that the vector field is irrotational.

   b) Define the divergence of a vector field and show that the vector $\vec{A} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$ is solenoidal.

8. a) Using Laplace transform, solve $\frac{d^2y}{dt^2} - 4y = 24\cos 2t$, given that $y(0) = 3$, $y'(0) = 4$.

   b) Find the following:

   i) $L\left\{e^{-3t}\cos 4\right\}$ and   ii) $L^{-1}\left\{\frac{3s+5}{s^2-2s-3}\right\}$

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