Roll No .....

MA-110

B.E. (All Branches), I Year I Semester

Examination, December 2015

Choice Based Credit System (CBCS)

Mathematics - I

Time : Three Hours

Maximum Marks: 60

Note: Attempt any five questions. All questions carry equal marks.

1. a) If $y = \sin(m \sin^{-1}x)$, 3

   prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.

   b) The equation of the tangent at the point (2, 3) of the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b. 4

   c) Evaluate $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$. 5

2. a) Expand by Maclaurin's theorem $e^{x\cos x}$ as far as the term x^3. 3

   b) Prove that the curvature at the point (x, y) of the catenary

   $y = c\cosh\left(\frac{x}{c}\right)$ *is* $\frac{y^2}{c}$. 4

   c) Locate the stationary points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and determine their nature. 5

3. a) If $u = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$, then prove that

   $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$. 3

   b) The radius of a sphere is found to be 10cm with a possible error of 0.02cm. What is the relative error in computing the volume? 4

   c) If $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, then show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)} = r^2\sin\theta$. 5

4. a) Evaluate $\lim_{n\to\infty}\left(\frac{1}{1+n^3} + \frac{4}{8+n^3} + \frac{9}{27+n^3} + \ldots + \frac{1}{2n}\right)$ 3

   b) Prove that $\int_{-\infty}^{\infty} e^{-a^2x^2}\, dx = \frac{\sqrt{\pi}}{a}$, $a > 0$. 4

   c) Express $\int_0^1 x^m(1-x^n)^p\, dx$ in terms of Beta functions and hence evaluate $\int_0^1 x^5(1-x^3)^{10}\, dx$. 5

5. a) Evaluate $\iint y\, dx\, dy$ over the part of the plane bounded by the line $y = x$ and the parabola $y = 4x - x^2$. 3

   b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\, dz\, dy\, dx$. 4

   c) Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. 5

6. a) Evaluate $\int_a^b e^x\, dx$ as limit of sum. 3

   b) Express in terms of the Gamma function : $\int_0^\infty x^n e^{-a^2x^2}\, dx$. 4

   c) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ and hence evaluate the same. 5

7. a) Verify Rolle's theorem, where $f(x) = 2x^3 + x^2 - 4x - 2$. 3

   b) If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 4

   c) Trace the curve $y^2(2a - x) = x^3$. 5

******