UNIT – 1

Pipe Flow, Pipe Network & Turbulent Flow

Unit-01/Lecture-01

Energy Losses in Pipes

When a fluid flows through a pipe, it experiences some resistance due to which the energy of fluid is lost. This loss of energy may be of two types:

1. **Major energy losses**
   These are due to friction and are calculated by Darcy-Weisbach formula and Chezy's formula.

   a. **Darcy-Weisbach formula:**
   Head loss due to friction is given by

   $$\mathbf{h_f = 4\ f\ L\ V^2\ /\ d\times 2g}$$

   where f = coefficient of friction
   = $16/R_e$ (for $R_e < 2000$, viscous flow)
   = $0.079 / (R_e)^{1/4}$ (for R_e varying from 4000 to 10^6)
   L = length of pipe, V = mean velocity of flow, d = diameter of pipe

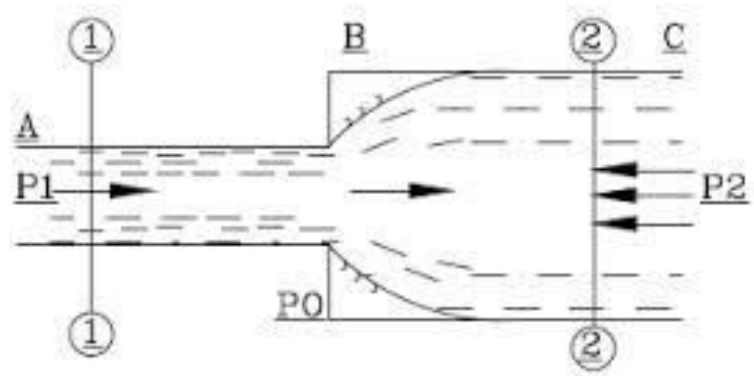
   b. **Chezy's formula:**

   $$\mathbf{V = C\ (mi)^{1/2}}$$

   Where $C = (\text{Þ}g/f')^{1/2}$ = Chezy's constant
   m = hydraulic mean depth for pipe flow = d/4
   $i = (h_f/L)^{1/2}$ = loss of head per unit length of pipe

2. **Minor energy losses**
   Loss of energy due to change of velocity is called the minor energy loss. It includes following cases:

   i. **Head loss due tosudden expansion**
   Consider a liquid flowing through a pipe having sudden enlargement. Let p_1, p_2 = pressure intensity at sections 1-1 and 2-2.
   V_1, V_2 = velocity of flow at sections 1-1 and 2-2
   A_1, A_2 = area of pipe at sections 1-1 and 2-2
   p' = pressure at liquid eddies on area (A_2- A_1) = p_1 (experimentally)
   h_e = head loss due to sudden enlargement.



Now,
force acting in the control volume of liquid between 1-1 and 2-2 in the direction of flow is given by

$F = p_1A_1 + p'(A_2-A_1) - p_2A_2$
$= p_1A_1 + p_1(A_2-A_1) - p_2A_2$
$= (p_1-p_2)\ A_2$ ……… (1)

Now,
Change of momentum of liquid/sec between the sections
$= ÞA_2V_2^2 - ÞA_1V_1^2$
$= ÞA_2\,(V_2^2 - V_1V_2)$ ……… (2)
(since from continuity equation, $A_1 = A_2V_2/V_1$)

Equating (1) and (2), we get

$(p_1 - p_2)/Þ = V_2^2 - V_1V_2$ …… (3)

Now,
Applying Bernoulli's equation between sections 1-1 and 2-2, we get
$h_e = (p_1 - p_2)/Þg + (V_1^2 - V_2^2)/2g$

Hence, from eq. (3) we get,

$$\mathbf{h_e = (V_1 - V_2)^2\ /\ 2g}$$

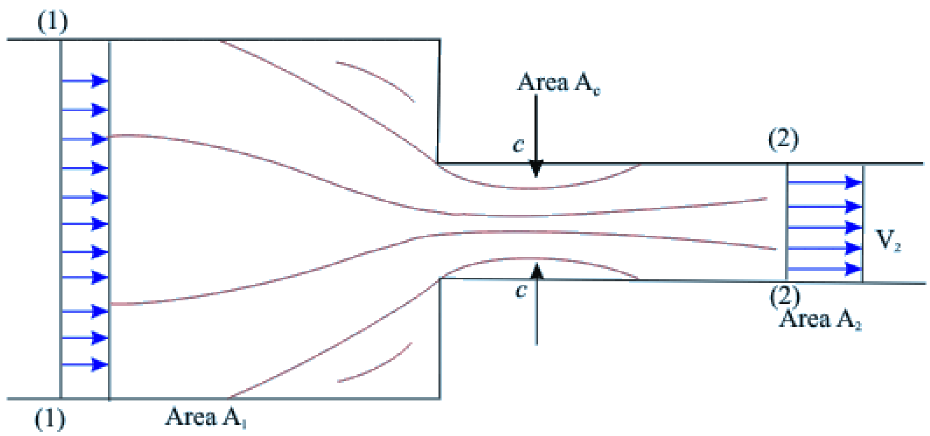
S.No.	RGPV questions	Year	Marks
Q.1	Derive an expression for the head loss due to sudden expansion in pipe flow.	Dec2010, June 202	7

Unit-01/Lecture-02

ii. **Head Loss due to sudden Contraction**

When liquid flows from large pipe to small pipe, the area of flow becomes minimum at section C-C, which is called Vena-contracta.

Let A_c, V_c be the area and velocity of flow at section C-C. andh_c be the head loss due to sudden contraction, which is actually due to sudden enlargement from section C-C to section 2-2.



Hence

$h_c=(V_c - V_2)^2/ 2g = V_2^2 [(V_c/V_2)- 1]^2/2g$

but $V_c/V_2 = A_2/A_c = 1/C_C$

hence **head loss due to sudden contraction** is given by-

$$h_c = V_2^2 [(1/C_C)- 1]^2/2g$$

or $h_c = k V_2^2/ 2g$

if C_C is assumed to be 0.62, then

$h_c = 0.375 V_2^2/ 2g$

when C_c is not given, then the **head loss due to sudden contraction** is taken as

$$h_c = 0.5 V_2^2/ 2g$$

iii. **Head loss at the entrance of a pipe**

It occurs when a liquid enters a pipe which is connected to a large tank or reservoir. It is similar to the head loss due to sudden contraction. It depends upon the form of entrance. For a sharp edge entrance, it is slightly more than a rounded or bell mouthed entrance.

$$h_{entrance}= 0.5 V^2/ 2g$$

iv. **Head loss at the exit of pipe**

It is due to the velocity of liquid at the outlet of the pipe which is dissipated either in the form of free jet or it is lost in the tank or reservoir.

$$h_{exit} = V^2/ 2g$$

v. **Head loss due to obstruction in a pipe**

It takes place due to reduction of area of the cross section of the pipe when there is any obstruction in the pipe.

$$h_o = V^2\left\{\frac{A}{Cc\,(A-a)} - 1\right\}^2/2g$$

Where a = Maximum area of obstruction, A = Area of pipe, V= Velocity of liquid in pipe and C_c = Coefficient of contraction.

vi. **Head loss due to bend in pipes and various pipe-fittings**

When there is any bend in pipe, the velocity of flow changes due to which separation of the flow from the boundary and also formation of eddies takes place.

$$h_b \text{ or } h_p = kV^2/ 2g$$

Where k = coefficient of bend or pipe fitting.

Hydraulic Gradient Line (HGL) and Total Energy Line (TEL)

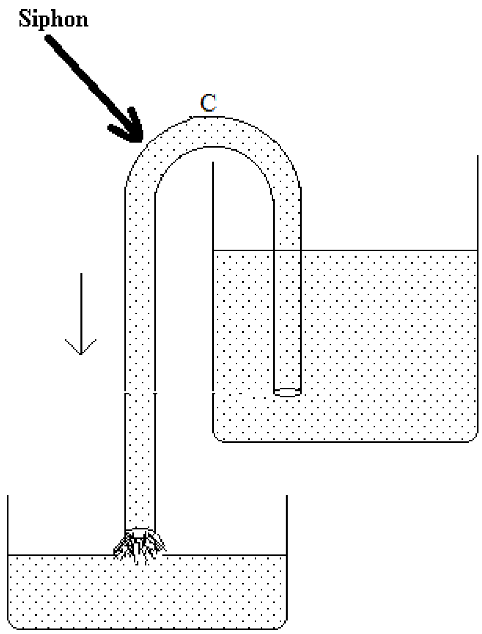
1. **HGL:** It isthe line which gives the sum of pressure head (p/w) and datum head (z) of a flowing fluid in a pipe with respect to some reference line. It is obtained by joining the tops of all vertical ordinates showing the pressure head of a flowing fluid in a pipe from the centre of the pipe.

2. **TEL:** It is the line which gives the sum of pressure head (p/w), datum head (z) and kinetic head ($V^2/2g$) of a flowing fluid in a pipe with respect to some reference line. It is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.

Unit-01/Lecture-03

Syphon

It is a longbent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at lower elevation when the two reservoirs are separated by a hill or high level ground.



The point C in the syphon is known as Summit. At the summit, negative pressure is created due to which liquid comes from the upper reservoir to the summit which is relatively at higher level from the upper reservoir.

Syphon is used in following cases:

1. To take out liquid from a tank not having any outlet.
2. To empty a channel not provided with any outlet sluice.
3. To carry water from one reservoir to another reservoir separated by a hill or ridge.

Pipes in Series (compound pipes)

Pipes in series is defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line (as shown in figure).

The discharge through each pipe is same. Hence,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in pipes. Hence,

$$\mathbf{H = h_{entrance} + h_{f1} + h_C + h_{f2} + h_e + h_{f3} + h_{exit}}$$

Unit-01/Lecture-04

Equivalent Pipe

It is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is known as Equivalent Size of the pipe. The length of equivalent pipe is equal to the sum of lengths of the compound pipe consisting of several pipes (previous figure). Hence,

$$L = L_1 + L_2 + L_3$$

Total head loss in the compound pipe (neglecting the minor losses),

$$H = h_{f1} + h_{f2} + h_{f3}$$

If $f_1 = f_2 = f_3 = f$, and $Q = \pi d_1^2 V_1/4 = \pi d_2^2 V_2/4 = \pi d_3^2 V_3/4$

$$\text{Or } V_1 = 4Q/\pi d_1^2,\ V_2 = 4Q/\pi d_2^2 \text{ and } V_3 = 4Q/\pi d_3^2$$

Hence,

$$H = 4\times 16fQ^2[L_1/d_1^5 + L_2/d_2^5 + L_3/d_3^5] / / \pi^2\ 2g \ .....(1)$$

But head loss in equivalent pipe due to friction,

$$H = 4fLV^2/d\times 2g = 4fL\,(4Q/\pi d^2)^2 / d\times 2g \ .....(2)$$

Hence, equating (1) and (2), we get

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

The above equation is called **Dupuit's equation**.

Pipes in parallel

The discharge through the main pipe is increased by connecting the pipes in parallel.

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence,

$$\mathbf{Q = Q_1 + Q_2}$$

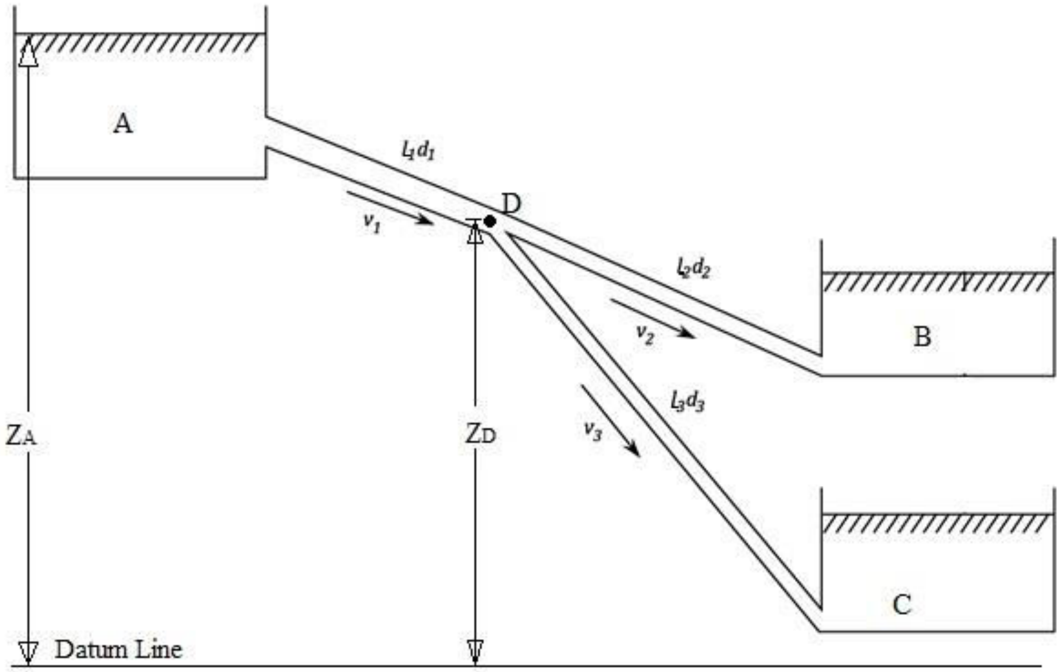
Also. The head loss for each branch pipe is same.

$$\mathbf{h_{f1} = h_{f2}}$$

Flow through branched pipes

When three or more reservoirs are connected by means of pipe having one or more junctions, the system is called branched pipes.

It is assumed that reservoirs are very large and the water surface levels are constant so that steady conditions exist in the pipes. Also minor losses are assumed to be very small.



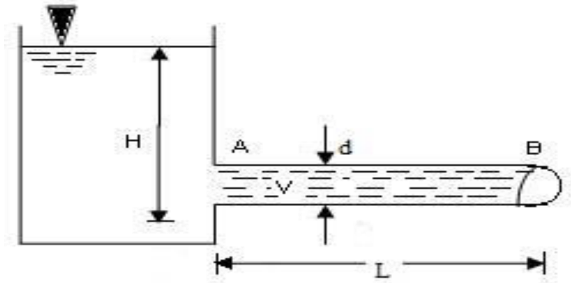
The flow from the junction D towards reservoir B will take place only when piezometric head at D ($p_D/w + Z_D$) is more than piezometric head at B (Z_B).

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Explain the term equivalent pipe. Also explain the syphon.	Dec 2009	7

Unit-01/Lecture-05

Power transmission through pipes

The power transmitted depends upon (i) the weight of liquid flowing through the pipe and (ii) the total head available at the end B of the pipe.



The power transmitted at the outlet of the pipe = weight of water flowing through the pipe per second (W) × head available at outlet

Or

$$P = (\text{Þ}g\times \pi d^2/4 \times V) \times (H - h_f) \text{ Watts}$$

Where area = $\pi d^2/4$, velocity = V and $h_f = 4fLV^2/2gd$

Efficiency of power transmission

η = Power at outlet of the pipe/power at inlet of the pipe

Or

$$\eta = W\,(H - h_f) / W\,H$$

$$\eta = (H - h_f) / H$$

- Condition for maximum power

  $\frac{d(P)}{dV} = 0$

Hence by using the value of h_f and putting the value of P in above equation , we get,

$$\mathbf{H = 3h_f}$$

- Maximum efficiency of transmission of power

$$\eta = (H - h_f)/H$$

Putting $h_f = H/3$, we get

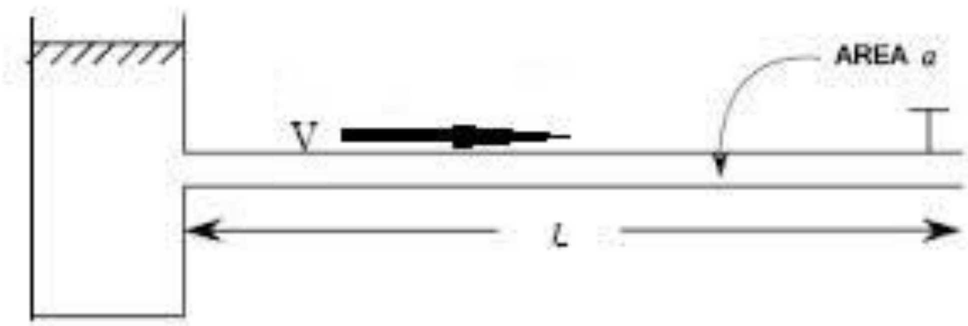
$$\mathbf{\eta = 2/3 \text{ or } 66.7\ \%}$$

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Show that for maximum transmission of power by means of pipe under pressure, the frictional loss of head in the pipe equals one third of the total head supplied. Also prove that the maximum power is restricted to 66.67%.	Dec 2011	7

UNIT 1/LECTURE 6

Water hammer in pipes

Consider a valve is connected to a long pipe AB as shown in figure to regulate the flow of water. When the valve is completely open, water flows with a velocity V in the pipe. If the valve is suddenly closed, the momentum of the flowing water will be destroyed and a wave of high pressure will be set up.this wave will be transmitted to along the pipe with a velocity of sound wave and may create noise called *knocking*. Also it has the effect of hammering action on the walls of the pipe, and hence it is also called as **Water Hammer**.



The following cases will be considered:

1. Gradual closure of valve

   Let t be the time in seconds to close the valve and p be the intensity of pressure wave produced.

   Force due to pressure wave = Mass of water in pipe × Retardation of water

   pA = ÞAL × (V-0)/t

   or p = ÞLV/t

   The above equation is valid for incompressible fluids and when pipe is rigid.

   Head of pressure, H = p/Þg = LV/gt

   If t > 2L/C, the valve closure is said to be gradual.

   If t < 2L/C, the valve closure is said to be sudden.

   Where C = velocity of pressure wave.

2. Sudden closure of valve and pipe is rigid

   Let K is the bulk modulus of water. When the valve is closed suddenly, the kinetic energy is converted into strain energy of water if the effect of friction is neglected and pipe wall is assumed perfectly rigid.

   Hence,

Loss of kinetic energy = Gain of strain energy

$\frac{1}{2} \times mass \times V^2 = \frac{1}{2} \times (p^2/K) \times Volume$

$\frac{1}{2} \times ÞAL \times V^2 = \frac{1}{2} \times (p^2/K) \times (AL)$

Or

$$\mathbf{p = V(KÞ)^{1/2}}$$

or

$$\mathbf{p = ÞV \times C}$$

3. Sudden closure of valve and pipe is elastic

Let t = thickness of the pipe wall,

E = modulus of elasticity of the pipe material,

1/m = Poisson's ratio for pipe material,

p = increase of pressure due to water hammer,

D = diameter of the pipe.

Total strain energy stored in pipe material = strain energy per unit volume × total volume

$= (p^2D^2/8Et^2) \times (\pi\ D\ t\ L) = p^2A\ DL/2Et$

Now,

Loss of kinetic energy of water = Gain of strain energy in water + Total strain energy stored in the pipe material

or

$\frac{1}{2} \times ÞAL \times V^2 = \{\frac{1}{2} \times (p^2/K) \times (AL)\} + (p^2A\ DL/2Et)$

Or

$$\mathbf{p = V\left[\frac{Þ}{\frac{1}{k}+\frac{D}{Et}}\right]^{1/2}}$$

UNIT 1/LECTURE 7

Hardy Cross Method

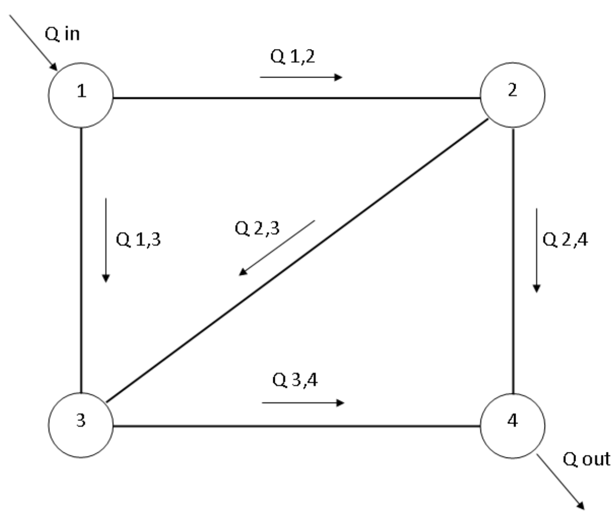
It is a method of successive approximations for analysis of pipe-network problems. The procedure is as follows:

1. A trial distribution of discharges is made arbitrarily but in such a way that continuity equation is satisfied at each junction or node.
2. With the assumed values of Q, the head loss in each pipe is calculated by following equation.

   $h_f = rQ^2$ (where $r = 4fL/2g \times (\pi/4)^2 \times D^5$)
3. In any loop, the algebraic sum of head losses round each loop must be zero. It means that in each loop, head loss due to flow in clockwise direction must be equal to the head loss due to flow in anticlockwise direction.
4. The net head loss is calculated around each loop. If the net head loss due to assumed values of Q round the loop is zero, then the assumed values of Q in that loop are correct. But if it is not zero, then the assumed values of Q are corrected by introducing a correction ΔQ for the flows till the circuit is balanced. The correction factor is obtained by:

   $$\Delta Q = -\sum r\, Q_0^{n} / \sum nrQ_0^{n-1}$$

   For turbulent flow, n = 2.
5. If the value of ΔQ comes out to be positive, then it should be added to the flows in the clockwise direction and subtracted from the flows in the anticlockwise direction.
6. Some pipes may be common to two circuits (or loops), then the two corrections are applied to these pipes.
7. After the corrections have been applied to each pipe in a loop, a second trial calculation is made for all loops. The procedure is repeated till ΔQ becomes negligible.



UNIT 1/ LECTURE 8

Boundary Layer and Boundary Layer Theory

When a solid body is immersed in a flowing fluid, there is a narrow region of the fluid in the neighbourhood of the solid body, where the velocity of fluid varies from zero to free stream velocity. This narrow region is called Boundary Layer.

According to boundary layer theory, the flow of fluid in the neighbourhood of the solid boundary can be divided into two regions:

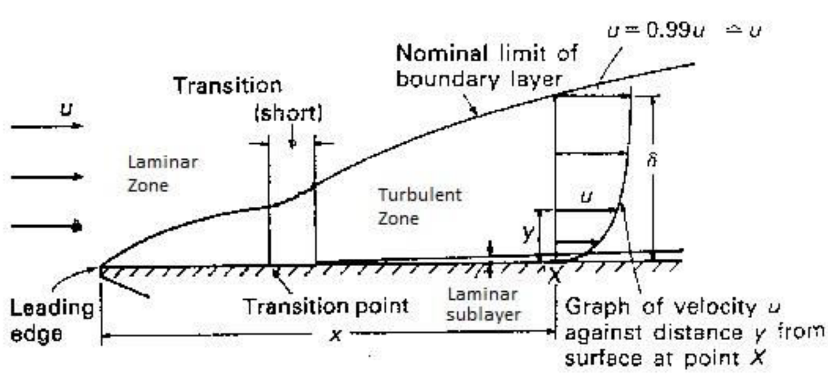
1. A very thin layer of fluid, called the boundary layer, in the neighbourhood of the solid boundary, where the variation of velocity from zero at the boundary to free stream velocity in the direction normal to the boundary takes place. In this region, the velocity gradient du/dy exists and hence the fluid exerts a shear stress on the wall in the direction of motion.
2. The remaining fluid, which is outside the boundary layer. The velocity outside the boundary layer is constant and equal to free stream velocity. As there is no variation of velocity in this region, the velocity gradient du/dy becomes zero. Hence, the shear stress is zero.

Laminar and Turbulent Boundary Layer

The boundary layer is called **laminar boundary layer** if the Reynold's number of the flow, which is given by $R_e = Ux/v$, is less than 5×10^5.

Where U = free stream velocity of flow, x = distance from leading edge and v = kinematic viscosity of fluid.

BOUNDARY LAYER ON FLAT PLATE



The length of the plate from the leading edge, up to which laminar boundary layer exists, is called laminar zone.

If the Reynold's number is more than 5×10^5 beyond the transition zone, the boundary layer is said to be **Turbulent Boundary Layer.**

Laminar Sub-layer

It is the region in the turbulent boundary layer zone adjacent to the solid surface of the plate. In this zone, velocity variation is influenced only by viscous effects. The velocity distribution is a parabolic curve in the laminar sub-layer zone, but in view of very small thickness, it is assumed that the velocity variation is linear. Hence, the shear stress in the laminar sub-layer zone is constant and equal to boundary shear stress.

Boundary Layer Thickness (δ)

It is the distance from the boundary of the solid body measured in y-direction to the point where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity (U) of the fluid. Following three such definitions are commonly adopted.

Displacement Thickness (δ*)

It is the distance measured perpendicular to the boundary of the solid body by which the boundary surface would have to be displaced outwards so that the total actual discharge would be same as that of an ideal fluid past the displaced boundary or to compensate for the reduction in flow rate on account of boundary layer formation.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Momentum Thickness (θ)

It is the distance measured perpendicular to the boundary by which the boundary should have to be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Energy Thickness (δ)**

It is the distance measured perpendicular to the boundary by which the boundary should have to be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation.

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

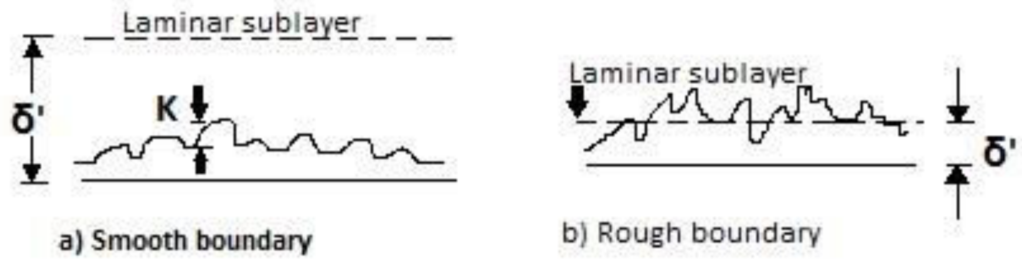
Hydro-dynamically Smooth and Rough Boundaries

Let K is the average height of the irregularities on the surface of a boundary. If the value of K is large, then the boundary is said to be Rough and if the value of K is small, then the boundary is said to be Smooth.

Outside the laminar sublayer, the flow is turbulent and eddies of various sizes present in the turbulent flow try to penetrate the laminar sublayer and reach to the surface of the boundary. But due to great thickness of laminar sublayer, eddies are unable to reach the surface irregularities and hence the boundary behaves as a smooth boundary. This type of boundary is called **hydro-dynamically smooth boundary**. If the thickness of the laminar sublayer (δ’) is much smaller than the average height K of the irregularities, the boundary is said to be **hydro-dynamically rough boundary**.

From Nikuradse’s experiment:

1. If **K/δ’ < 0.25**, the boundary is called **Smooth boundary**.
2. If **K/ δ’ > 6**, the boundary is called **Rough boundary**.



In terms of Reynold's number (u_*K/ν)

1. If $u_*K/\nu < 4$, the boundary is considered as **Smooth**.
2. If $u_*K/\nu > 100$, the boundary is considered as **Rough**.

Where

u_* = Shear velocity of fluid in pipe flow = $(\tau_0/\rho)^{1/2}$

τ_0 = Shear stress in pipe flow = $(p_1 - p_2)d/4L$

$(p_1 - p_2)$ = pressure drop in pipe flow between two sections

d = diameter of pipe

L = length of pipe

ν = Kinematic viscosity of fluid

S.NO	RGPV QUESTION	YEAR	MARKS
Q.1	What do you understand by the terms boundary layer and boundary layer theory.	June 2012	7
Q.2	Explain the development of boundary layer along a thin and smooth plate held parallel to uniform flow. Point out the salient features.	Dec 2010	7
Q.3	Define physically and mathematically the concept of displacement, momentum and energy thickness of a boundary layer.	Dec 2011, Dec 2009	7

Q1) A compound pipe system consists of 1800 m of 50 cm, 1200 m of 40 cm & 600 m of 30 cm pipe of the same material connected in series.

(i) What is the equivalent size at a pipe 3600 m long?

(ii) What is the equivalent length at 40 cm pipe at the same material?

[RGPV June 2012]

Soln Let the coefficient of friction is same for all pipes.

And $L = 3600 = L_1 + L_2 + L_3 = 1200 + 600 + 1800$

Hence by Dupuit's equation

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

(i) for $L_e = 3600$ m,

$$\frac{3600}{D_e^5} = \frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}$$

$\therefore \quad D_e = 0.386 \text{ m} \approx$ 39 cm.

(ii) for $D_e = 40$ cm,

$$\frac{L_e}{(0.4)^5} = \frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}$$

$L_e = 4318$ m

Q.2. Determine whether the pipe will act as hydrodynamically smooth, in transition or rough in the following cases:

(i) $D = 300$ mm, $L = 50$ m, drop in pressure $= 4.2$ kN/m²
$K = 0.02$ mm, $\rho = 998$ kg/m³, $\nu = 10^{-6}$ m²/s

(ii) $\tau_0 = 638.78$ N/m², $\rho = 998$ kg/m³, $\nu = 10^{-6}$ m²/s,
$K = 2$ mm for rivetted steel pipe.

[RGPV Dec 2009]

Soln (i) $\tau_0 = (p_1 - p_2)\cdot\frac{D}{4L} = (4.2\times10^3)\cdot\frac{0.30}{4\times50} = 6.3\ N/m^2$

Shear velocity, $u_\star = \sqrt{\tau_0/\rho} = \sqrt{6.3/998}$

$= 0.0794\ m/s$

Now $\frac{u_\star \cdot K}{\nu} = \frac{0.0794 \times 0.02\times10^{-3}}{10^{-6}} = 1.59$

$\because \frac{u_\star K}{\nu} < 4$, Hence the boundary is Smooth.

(ii) $u_\star = \sqrt{\tau_0/\rho} = \sqrt{638.78/998} = 0.8\ m/s$

Now $\frac{u_\star K}{\nu} = \frac{0.8\times 2\times10^{-3}}{10^{-6}} = 1600\ m/s$

$\because \frac{u_\star K}{\nu} > 100$, the boundary is Rough.