UNIT – 2

Uniform Flow in Open Channels

Lecture-01

Introduction & Definition

Open-channel flow, a branch of hydraulics, is a type of liquid flow within a conduit with a free surface, known as a channel. The other type of flow within a conduit is pipe flow. Open-channel flow has a free surface, whereas pipe flow does not. {Modi & Seth}

Difference between Open channel flow & Pipe flow (RGPV June 2012) {Modi & seth}

1.) Open channel flow must have a free surface but pipe flow does not.
2.) Flow in open channel takes place under gravity force & pressure is atmospheric. Whereas pipe flow contains hydraulic pressure only.
3.) In open channel, cross-section may be of any shape (rectangular, circular, trapezoidal etc.) but in pipe flow, cross section is usually circular.
4.) Velocity distribution is different. In pipe flow, maximum velocity is at centre but in open channel flow, maximum velocity occurs at 10-25 % depth from the top.

Types of open channel flows {R K Bansal}

1.) Steady flow & Un-steady flow:

If the flow characteristics such as depth, velocity, rate of flow at any point do not change wrt time, the flow is called Steady flow. Otherwise it is Un-steady flow.

2.) Uniform flow & Non-uniform flow:

If for a given length of channel, the velocity, depth, slope of channel & cross section remains constant, it is called uniform flow. Otherwise, it is non-uniform flow.

3.) Rapidly varied flow (RVF) & Gradually varied flow (GVF):

RVF is the flow in which depth changes abruptly over a small length of channel. When there occurs any obstruction in the path of flow, water level rises over the obstruction & then falls & again rises over a small length of channel. Thus depth of flow changes rapidly over a short length of the channel.
GVF is the flow when depth of flow changes gradually over a long length of channel. This condition

occurs before any large obstruction in the path of the flow.

4.) Laminar & Turbulent flow:

The flow is said to be laminar if the Reynolds number is less than 500 or 600. If the Reynolds number (R_e) is greater than 2000, the flow is said to be turbulent. Between 500-2000, the flow is in transition state.

$$R_e = \rho VR/ \mu$$

Where V = mean velocity of flow,
R = hydraulic mean depth or Hydraulic radius, (R = A/P)
Þ and μ= density and viscosity of water

5.) Subcritical, Critical & Supercritical flow:

Flow is subcritical if the Froude number (F_e) is less than 1.
Flow is critical if $F_e = 1$.
Flow is supercritical if F_e is more than 1.

The Froude no. is defined as $F_e = V/ (\sqrt{gD})$
Where D hydraulic depth of channel. D = A/T.

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Differentiate open channel flow & pipe flow in detail.	June 2012, Dec 2012	7, 4
Q.2	Distinguish between subcritical flow & supercritical flow.	Dec 2014, Dec 2011	2, 3
Q.3	Differentiate between i) steady & unsteady flow, ii) uniform & non-uniform flow	June 2009	10

Unit-02/Lecture-02

Geometrical Elements of Channel Section

1.) **Depth of flow (y):** Vertical distance from the lowest point of the channel from the free surface.

2.) **Top width (T):** Width of the top of the channel section.

3.) **Wetted perimeter (P):** Length of the channel section on contact with the flowing water at any section.

4.) **Wetted area (A):** Area of flow section normal to the direction of flow.

5.) **Hydraulic radius or Hydraulic mean depth (R):** Ratio of wetted area to its wetted perimeter.

$$R = A/P$$

6.) **Hydraulic depth (D):** Ratio of wetted area to the top width of the channel section.

$$D = A/T$$

7.) **Section factor (Z):** it is required for the computation of critical flow.

$$Z = (A^3/T)^{1/2}$$

Velocity distribution in open channel

- The velocity of flow at any channel section is not uniformly distributed. It is due to the presence of a free surface & frictional resistance along the channel boundary.
- The velocity distribution in a channel is measured either with the help of a pitot tube or a current meter.
- The mean velocity of flow is computed from vertical velocity distribution curve obtained by actual measurements. The velocity at 0.6 depth from the free surface is very close to the mean velocity. For better approximation, average of velocities at 0.2 depth & 0.8 depth from the free surface is taken.

$$V_{mean} = (V_{0.2} + V_{0.6})/2$$

- Maximum velocity occurs at 0.1 to 0.25 depth from the free surface. The velocity at top is around 90% of the maximum velocity.

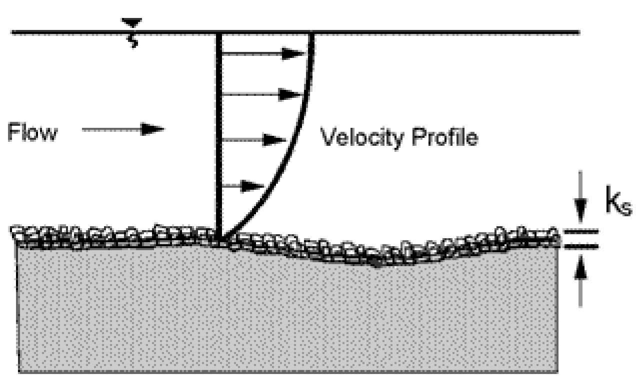


Figure 1: Velocity distribution in Open Channel

	RGPV QUESTIONS	Year	Marks
Q.1	Define hydraulic radius, wetted perimeter, gradually varied flow and rapidly varied flow.	Dec 2009	7
Q.2	Define the following: 1. Hydraulic radius 2. Hydraulic depth 3. Froude number 4. Specific Energy	June 2012, Dec 2012	7, 4
Q.3	Define hydraulic radius, wetted perimeter.	Dec 2014	2

Unit-02/Lecture-03

Specific Energy [RGPV June 2012]

If the channel bottom is taken as datum, the total energy per unit weight of liquid will be

$$E = y + V^2/2g$$

Where E is known as specific energy which is defined as energy per unit weight of liquid wrt the channel bottom.

Specific Energy Curve: It is the curve showing the variation of specific energy with depth of flow.

$$E = y + V^2/2g = E_p + E_k$$

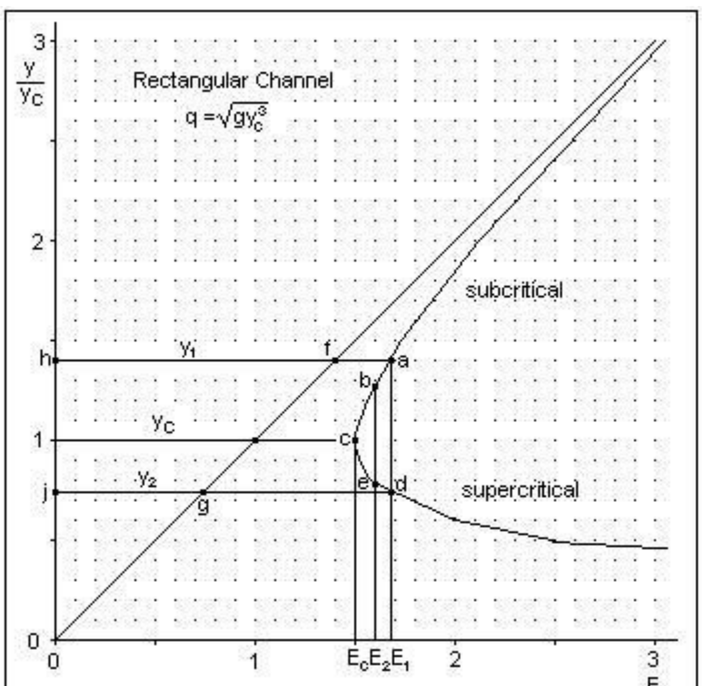
Where E_p& E_k are Potential and Kinetic energy of flow.
Consider a rectangular channel in which steady but non-uniform flow is taking place.

Velocity of flow, $V = Q/A = Q/by = q/y$.

Where
Q = discharge through the channel,
b = width of channel, h = depth of flow,
q = discharge per unit width. Then q is constant.
Hence,

$$E = y + q^2/2gy^2 = E_p + E_k$$

From above equation, graph between E (x-axis) and y (y-axis) is plotted, which will be the specific energy curve.



Specific Energy Diagram

Unit-02/Lecture-04

Critical depth (y_c): depth of flow of water at which the specific energy is minimum.

$$E = y + q^2/2gy^2$$

For E to be minimum

$$dE/dy = 0$$

it gives

$$\mathbf{y = (q^2/g)^{1/3} = y_c}$$

Critical velocity (V_c): It is defined as the velocity of flow at the critical depth. It is obtained as following.

$$q = \text{Discharge per unit width}$$

$$q = Q/b = AV/b = byV/b = yV = y_cV_c$$

Putting in the expression of critical depth, we will get

$y_c = (q^2/g)^{1/3} = (y_c^2\ V_c^2 / g)^{1/3}$

or

$$\mathbf{V_c = (g\ y_c)^{1/2}}$$

Critical flow: It is defined as the flow at which the specific energy is minimum or it is the flow corresponding to the critical depth.

We have

$$V_c = (g\ y_c)^{1/2}$$

Or

$$V_c/ (g\ y_c)^{1/2} = 1$$

Or

$$\mathbf{F_e = 1}$$

Hence, critical flow occurs when Froude number is unity.

Critical depth in terms of Minimum specific energy

Specific energy is minimum when depth of flow is critical and hence

$$E_{min} = y_c + q^2/2gy_c^2$$

Putting the expression for critical depth (y_c) in the above, we have

$$E_{min} = y_c + y_c/2 = 3y_c/2$$

Or

$$\mathbf{y_c = \frac{2}{3} (E_{min})}$$

Hence, critical flow occurs when the depth of flow is 2/3rd of the specific energy.

Alternate depths & Conjugate depths

In the specific energy curve, at any other value of specific energy than the minimum value, there are two depths of flow (y_1 & y_2) for which a given discharge may occur with same specific energy. They are called ***Alternate Depths*** or *Sequent depths*.

In case of hydraulic jump (non-uniform flow condition), a considerable amount of energy is lost. In such case Specific force analysis is done (Unit 3). In this case,, at any other value of specific force other than the minimum value, there are two similar depths of flow at same specific force. They are called ***Conjugate depths***.

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	What is a specific energy curve? What do you understand by critical depth of an open channel when the flow in it is not uniform?	Dec 2009, June 2015	10, 3
Q.2	What is critical flow? Show that critical flow occurs in an open channel when (i) Froude number is unity (ii) when the depth of flow is 2/3rd of the specific energy.	Dec 2010, Dec 2013	10, 7
Q.3	Distinguish between Alternate depth & conjugate depth.	June 2009	2

Unit-02/Lecture-05

Uniform flow and its computation

The computation of discharge for uniform flow is done by **Chezy's formula [RGPV Dec 2012]**, which is given by

$$Q = A\,V = A\,C\,\sqrt{R\,S}$$

Where V = mean velocity of flow, A = area of flow of water, R = hydraulic mean depth, S = Slope of channel bed and C = Chezy's constant.

The value of C can be given by

1. **Manning's formula**: according to this,

$$C = (1/N)\,R^{1/6}$$

Where N = Manning's constant

2. **Bazin formula (in MKS units)** : according to this,

$$C = \frac{157.6}{1.81 + K/\sqrt{m}}$$

Where K = Bazin's constant and depends on the roughness of the surface of channel
m = hydraulic mean depth

3. **Ganguillet-Kutter Formula**: according to this,

$$\frac{23 + \left(\frac{0.00155}{i}\right) + \left(\frac{1}{N}\right)}{1 + \left(23 + \frac{0.00155}{i}\right) N/\sqrt{m}}$$

Where N = Roughness coefficient which is called Kutter's constant,
i = slope of the bed,
m = hydraulic mean depth

Economical Sections of Channels

A channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction depends upon the excavation and lining. To keep the cost minimum, the wetted perimeter, for a given discharge, should be minimum.

Most Economical Rectangular Channel

Consider a rectangular channel of width b & depth of flow is y.
Hence area of flow,

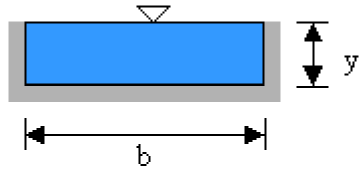
$$A = by$$

& Wetted Perimeter,

$$P = b + 2y$$

Or

$$P = (A/y) + 2y$$



For most economical section,

$$\frac{dP}{dy} = 0$$

By substituting, we get

$$A = 2y^2$$

But A = by, hence

$$\mathbf{b = 2y}$$

Now, hydraulic mean depth, R = A/P
Or

$$R = 2y^2/(2y+2y)$$

Or

$$\mathbf{R = y/2}$$

Hence, Conditions for most economical rectangular sections:

- Width is two times the depth of flow.
- Hydraulic mean radius is half the depth of flow.

Normal slope & Critical slope

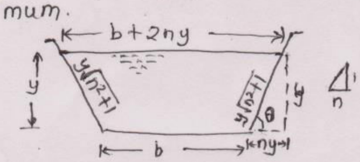
Normal slope (i_n): It is the slope of channel bottom which will maintain the uniform flow at a given depth of flow (slope in the Manning's formula).
Critical slope (i_c): It is the slope of channel bottom which is required to maintain uniform flow at critical depth (minimum specific energy condition).

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	State the Chezy's, Bazin, Kutter and Manning's formula for uniform flow through a channel. What are the dimensions of constant C in Chezy' formula.	Dec 2011, June 2015	10, 2
Q.2	What do you mean by most economical section of the channel? Explain the significance of most economical channel.	Dec 2014,	3
Q.3	Distinguish between Normal slope & Critical slope.	June 2009, June 2015	2

UNIT 2/LECTURE 6

Most Economical Trapezoidal section

The trapezoidal section of a channel is most economical if its perimeter is minimum.



Let b = width of channel bottom
y = depth of flow
θ = angle made by side with horizontal
& side slope is 1 Vertical to n horizontal.

Top width, $T = b + 2ny$

Now,
area of flow, $A = \frac{1}{2}\cdot(b+T)y = \frac{1}{2}(b+b+2ny)y = \underline{(b+ny)y}$ — ①

or $b = \frac{A}{y} - ny$ — ②

Perimeter (wetted), $P = b + 2y\sqrt{n^2+1}$

from ②, $P = \frac{A}{y} - ny + 2y\sqrt{n^2+1}$ — ③

for P to be minimum, $\frac{dP}{dy} = 0 = \frac{-A}{y^2} - n - 2\sqrt{n^2+1}$

or $\frac{A}{y^2} + n = 2\sqrt{n^2+1}$

from ①, $\frac{(b+ny)y}{y^2} + n = 2\sqrt{n^2+1}$

$\Rightarrow b + 2ny = 2y\sqrt{n^2+1}$ or $\boxed{T = 2y\sqrt{n^2+1}}$

* Hydraulic Mean depth (m):

$$m = \frac{A}{P} = \frac{(b+ny)y}{b+2y\sqrt{n^2+1}}$$

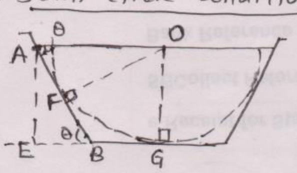
$$= \frac{(b+ny)y}{b+b+2ny} \Rightarrow \boxed{m = \frac{y}{2}}$$

* Side Angle (θ): θ depends on slope, n. Hence, n is variable.
for P to be minimum, $\frac{dP}{dn} = 0$. Hence differentiating eq. ③ wrt n

$$\Rightarrow \frac{dP}{dn} = \frac{d}{dn}\left[\frac{A}{y} - ny + 2y\sqrt{n^2+1}\right] = 0$$

$$\Rightarrow 2n = \sqrt{n^2+1} \quad \text{or} \quad 4n^2 = n^2+1 \quad \text{or} \quad \boxed{n = \frac{1}{\sqrt{3}}} \Rightarrow \boxed{\theta = 60^\circ}$$

* Semi-circle condition: Draw $OF \perp AB$



$\sin\theta = \frac{OF}{OA}$ or $OA\cdot\sin\theta = OF$

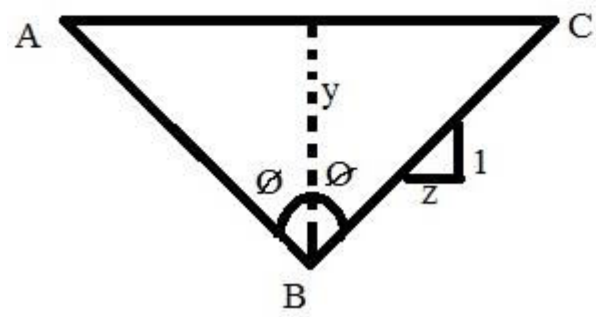
$$\Rightarrow OF = \sin 60^\circ\cdot\frac{(b+2ny)}{2} = \frac{\sqrt{3}}{2}\cdot\frac{2y\sqrt{n^2+1}}{2}$$

$$= \frac{\sqrt{3}}{2}\cdot y\cdot\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1} = \frac{\sqrt{3}}{2}\cdot y\cdot\frac{2}{\sqrt{3}} = y$$

Hence, $OF = OG = y$. Thus, a semi-circle can be drawn from O as centre & y as radius.

Most Economical triangular section

Consider a triangular channel with top width AC = T. let it has slopes in 1 in z.
Hence, T = 2zy.



Now, area of flow,

$$A = ½ T y = ½ (2zy) y$$

Or

$$A = zy^2$$

Or

$$y = (A/z)^{1/2}$$

Wetted perimeter, $P = 2y(z^2+ 1)^{1/2} = 2 (A/z)^{1/2} (z^2+ 1)^{1/2} = 2\sqrt{A} (z + 1/z)$
Now, for P to be minimum,

$$dP/dz = 0$$

It gives,

$$\mathbf{z = 1}$$

Now,

$$\tan \theta = zy/y = 1$$

Hence,

$$\boldsymbol{\theta = 45^0}$$

Hence, for most economical triangular section, the side slope should be 1 horizontal to 1 vertical. And the vertex angle is 90^0 (= 45^0 + 45^0).

Hydraulic mean depth, m = A/P
$A = y^2 \tan \theta$ & $P = 2y \sec \theta$, hence $m = (y \sin \theta)/2$
Putting the value of $\theta = 45^0$, we get

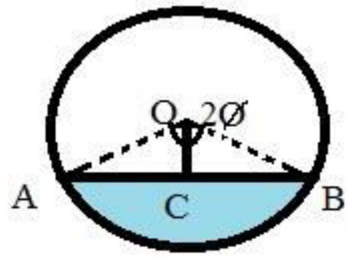
$$\mathbf{m} = \frac{y}{2\sqrt{2}}$$

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Show that for a hydraulically most efficient triangular section, the hydraulic radius: $R = y/2\sqrt{2}$	June 2012	7
Q.2	For a trapezoidal channel of most economical section, prove that i) Half the top width = length of side slope ii) Hydraulic mean depth = ½ depth of flow	Dec 2013	7

UNIT 2/LECTURE 7

Most Economical Circular Section

If 2Ø is the angle subtended by water surface and R is the radius of the channel.



Then, Wetted perimeter,

$$P = 2R\theta$$

And Wetted area,

$$A = R^2 \left(\theta - \frac{\sin 2\theta}{2}\right)$$

1. <u>Condition for maximum velocity</u>

$$\boldsymbol{\theta = 128^0 45' = 2.24 \text{ radians}}$$

and

Depth of flow = 0.81 D

where D = diameter of circular channel.

2. <u>Condition for maximum discharge</u>

$$\boldsymbol{\theta = 154^0 = 2.68 \text{ radians}}$$

And

Depth of flow = 0.95 D

UNIT 2/LECTURE 8

Q.1) A trapezoidal section has to be excavated in hard clay at minimum cost. Calculate the dimensions of channel to give a discharge of 15 m^3/sec at a slope of 1:2500. Take Manning's constant 0.02 [RGPV - Dec. 2009]

Soln: for trapezoidal section to be economical,

angle made by side slopes with horizontal, $\theta = 60°$

$\therefore \frac{1}{n} = \tan 60° = \sqrt{3}$ or $n = 1/\sqrt{3}$

Now, $\frac{b + 2nd}{2} = d\sqrt{n^2+1}$

Putting $n = 1/\sqrt{3}$, we get $b = 2d/\sqrt{3}$

Area of flow, $A = (b + nd)d = \left(\frac{2d}{\sqrt{3}} + \frac{d}{\sqrt{3}}\right)\cdot d = \sqrt{3}\, d^2$

H.M.D, $m = d/2$

Discharge, $Q = A \cdot V = A \cdot \left(\frac{1}{N} \times m^{2/3} \times i^{1/2}\right)$ (By Manning's formula)

$\Rightarrow 15 = (\sqrt{3}\, d^2)\left\{\frac{1}{0.02} \times \left(\frac{d}{2}\right)^{2/3} \times \left(\frac{1}{2500}\right)^{1/2}\right\}$

or $\boxed{d = 2.672 \text{ metres}}$

$\therefore b = \frac{2 \times 2.672}{\sqrt{3}} = 3.085$ metres.

$\Rightarrow \boxed{b = 3.085 \text{ metres}}$

Hence, width of the channel $(b) = 3.085$ m

& depth of the channel $(d) = 2.672$ m

Q.2) A trapazoidal channel is to carry 8 m^3/s of discharge at velocity of 1.5 m/sec. Design the most economical section if the channel has side slopes 1 vertical & 2 horizontal.

[RGPV Dec.2010]

Soln – Side slope, $n = \frac{\text{horizontal}}{\text{vertical}} = \frac{2}{1} = 2$

For most economical section,

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

Putting $n = 2$, we get. $b = 0.472\,d$

Now,

Area of flow, $A = (b + nd)\,d$

$$= [0.472d + 2d]\,d = 2.472\,d^2$$

But $A = \frac{Q}{V} = \frac{8}{1.5} = 5.333\ m^2$

$\therefore \quad 2.472\,d^2 = 5.333$

$\Rightarrow$ depth of flow, $d = \underline{1.468 \text{ metres}}$

Hence,

width of the channel, $b = 0.472\,d$

$$= 0.472 \times 1.468$$

$$= \underline{0.693 \text{ metres}}$$

UNIT 2/LECTURE 9

Q.3) A trapezoidal channel is required to carry $10 m^3/s$ water at a velocity of 2m/s. Design a most economical section if the section has side slopes 1 horizontal to 2 vertical. [RGPV, Dec 2009]

Soln: $n = \frac{1}{2} = 0.5$

For most economical trapezoidal section,

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

Putting $n = 1/2$, we get $b = 1.236\,d$

Now, area of flow, $A = \frac{Q}{V} = \frac{10}{2} = 5\ m^2$

But for trapezoidal section, $A = (b+nd)d$

$$= (1.236d + 0.5d)d$$

$$A = 1.736d^2$$

$\therefore \quad 1.736d^2 = 5$

$\Rightarrow \boxed{d = 1.697\ m}$ (depth of flow)

$\therefore \quad b = 1.236\,d = 1.236 \times 1.697$

$\Rightarrow \boxed{b = 2.097\ m}$ (width of channel)

Q.4) A trapezoidal channel having the side slope equal to $60°$ with the horizontal & laid on a slope of 1 in 800 carries a discharge of $10\ m^3/s$. Find the width at base & depth of flow for most economical section. Take $C = 60\ m^{1/2}/sec$. [RGPV, Dec 2011]

Soln Bed slope, $i = 1$ in $800 = 1/800$

Side slope, $\theta = 60°$

$\therefore \tan\theta = \tan 60° = \frac{1}{n} \Rightarrow n = 0.577$

Now, for most economical section,

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

$$\Rightarrow \frac{b+2(0.577)d}{2} = d\sqrt{(0.577^2)+1}$$

or $b = 1.155\, d$

Now $Q = A \cdot V = A \cdot C\sqrt{mi}$ (By Chezy's formula)

$$\Rightarrow 10 = (b+nd)d \times 60\sqrt{\frac{d}{2} \times \frac{1}{800}} \quad (\because m = \frac{d}{2})$$

Putting $n = 0.577$, we get
& $b = 1.155d$,

$$\boxed{d = 1.711 \text{ metres}}$$

Hence, depth of flow = 1.711 metres.

$\therefore$ Width of channel, $b = 1.155\, d$

$= 1.155 \times 1.711$

$$\boxed{b = 1.976 \text{ metres}}$$

Q.5) A rectangular channel 8 m wide & 1.5 m deep has a slope of 0.001 in 1 & is lined with concrete. It is required to increase the discharge to a maximum by changing the dimensions of channel but keeping the same amount of lining. Calculate the new dimensions & the percentage increase in discharge. Take Manning's constant for concrete as 0.015.

[RGPV Dec 2011]

Soln: Area of flow, $A_1 = b_1 \cdot d_1 = 8 \times 1.5 = 12 m^2$

Wetted perimeter, $P = b_1 + 2d_1 = 8 + 2(1.5) = 11 m$

Hyd. Mean depth, $R_1 = A/P = \frac{12}{11} = 1.09 m$

According to Manning's formula,

$$Q_1 = \frac{1}{N} \cdot A_1 \cdot R_1^{2/3} \cdot S^{1/2}$$

$$= \frac{1}{0.015} \cdot (12)(1.09)^{2/3}(0.001)^{1/2}$$

$$\boxed{Q_1 = 26.794 \ m^3/s}$$

For discharge to be maximum in rectangular channel,

$$b_2 = 2d_2$$

where b_2 & d_2 are new width & new depth of the section respectively.

Now, wetted perimeter, $P = b_2 + 2d_2 = 11$ (same)

$$\Rightarrow b_2 + b_2 = 11 \Rightarrow \boxed{b_2 = 5.5 m}$$

$$\therefore d_2 = \frac{11 - b_2}{2} = \frac{11 - 5.5}{2} = \boxed{2.75 m = d_2}$$

$\therefore$ New area of flow, $A_2 = 5.5 \times 2.75 = 15.125 m^2$

& New hyd. mean depth, $R_2 = \frac{15.125}{11} = 1.375 m$

Hence, new discharge

$$Q_2 = \frac{1}{N} \cdot A_2 \cdot R_2^{2/3} \cdot S^{1/2}$$

$$= \frac{1}{0.015} \times (15.125)(1.375)^{2/3}(0.001)^{1/2}$$

$$\boxed{Q_2 = 39.428 \ m^3/s}$$

Hence, percent increase in discharge $= \frac{Q_2 - Q_1}{Q_1} \times 100$

$$= \frac{39.428 - 26.794}{26.794} \times 100$$

$$\boxed{\text{\% increase in discharge} = 47.15 \%}$$

Ans