UNIT – 3

NON-UNIFORM FLOW IN OPEN CHANNELS

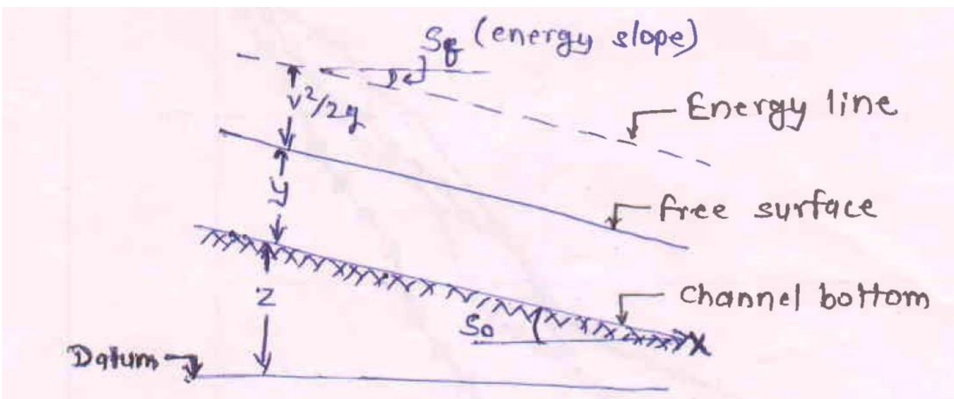
Lecture-01

Basic assumptions & Dynamic Equation of GVF

Assumptions:

1. Chezy's & Manning's formulae are applicable for determining the slope of the energy line or energy slope (S_f).
2. The channel bottom slope or bed slope (S_o) is small.
3. The flow is steady & hence the discharge is constant.
4. The channel is prismatic.
5. The energy correction factor is unity.
6. The roughness coefficient is constant for the length of the channel & it is independent of depth of flow.
7. The pressure distribution is hydrostatic (no acceleration effect).

Derivation:



Total energy head

$$E = z + y + v^2/2g$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) \text{...eq(1)}$$

Where

$\frac{dE}{dx}$ = -S_f (energy decreases in the direction of flow due to friction losses)

$\frac{dz}{dx}$ = - S_o

$\frac{dy}{dx}$ = slope of water surface wrt channel bottom or change of depth of flow in the direction of flow

$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{d}{dx}\left(\frac{q^2}{2gy^2}\right) = -\frac{q^2}{gy^3}.\frac{dy}{dx}$$

Hence eq. (1) becomes

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{q^2}{gy^3}}$$

Now

$$Q = q\,b = A\,V = (by)\,v$$

or

$$q = v\,y$$

hence

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{v^2}{g\,y}}$$

or

$$\boldsymbol{\frac{dy}{dx} = \frac{S_o - S_f}{1 - F_e^2}}$$

1. If $\boldsymbol{\frac{dy}{dx}}$ **=** 0, i.e. depth of flow is constant, it means that the water surface is parallel to the channel bed. (uniform flow case)
2. If $\boldsymbol{\frac{dy}{dx}} > 0$, then in the direction of flow, depth of flow will increase. The profile of water so obtained is called *Backwater Curve* or *Rising Curve*.
3. If $\boldsymbol{\frac{dy}{dx}} < 0$, then in the direction of flow, depth of flow will decrease. The profile of water so obtained is called *Drawdown Curve* or *Dropdown Curve* or *Falling Curve*.

S.No.	RGPV questions	Year	Marks
Q.1	Define GVF & RVF. Also give assumptions made in gradually varied flow.	Dec 2013, June 2015	4, 2
Q.2	Show that in a rectangular channel of constant width the slope of the water surface with respect to the bed line is given by $\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{v^2}{g\,y}}$ With usual notations.	Dec 2011	10
Q.3	What is back water curve & drawdown curve? Explain in details.	June 2009	5

Unit-03/Lecture-02

Dynamic eqn for GVF in Wide rectangular Channels

1. According to Chezy,

$$\frac{dy}{dx} = \frac{S_o\left\{1-\left(\frac{y_n}{y}\right)^3\right\}}{\left\{1-\left(\frac{y_c}{y}\right)^3\right\}}$$

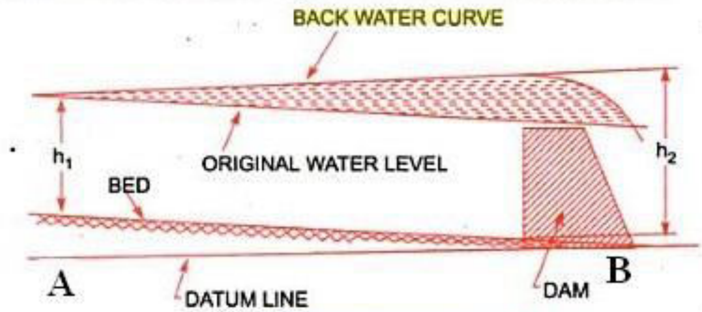
2. According to Manning,

$$\frac{dy}{dx} = \frac{S_o\left\{1-\left(\frac{y_n}{y}\right)^{10/3}\right\}}{\left\{1-\left(\frac{y_c}{y}\right)^3\right\}}$$

Where y_n = normal depth of flow (depth of uniform flow), y = actual depth of flow (depth of non-uniform flow), y_c = critical depth of flow.

Back water curve & Afflux

Back Water Curve and Affux. Consider the flow over a dam On the upstream side of the dam, the depth of water will be rising. If there had not been any obstruction (such as dam) in the path of flow of water in the channel, the depth of water would have been constant as shown by dotted line parallel to the bed of the channel Due to obstruction, the water level rises and it has maximum depth from the bed at some section.



Back water curve and affux.

Let h_1 = depth of water at the point, where the water starts rising up, (A) and
h_2 = maximum height of rising water from bed. (B)

Then $(h_2 - h_1)$ = Affux. Thus *affux* is defined as the maximum increase in water level due to dam as obstruction in the path of flow of water. The profile of the rising water on the upstream side of the dam is called *back water curve*. The distance along the bed of the channel between the section where water starts rising to the section where water is having maximum height is known as *length of back water curve*. (A to B)

Types of channel bottom slopes

1. Critical slope: when $S_o = S_c$ & $y_n = y_c$
2. Mild slope: when $S_o < S_c$ & $y_n > y_c$
3. Steep slope: when $S_c < S_o$ & $y_n < y_c$
4. Horizontal slope: when $S_o = 0$, y_n = infinite & channel bottom is horizontal.
5. Adverse slope: when $S_o < 0$, y_n is negative or imaginary or non-existent & channel bottom rises in the direction of flow.

Characteristics of surface profiles *(important for GATE)*

1. **In Mild sloped channels ($y_n > y_c$)**

Condition	**Surface Profile**
$y > y_n > y_c$	**M_1**
$y_n > y > y_c$	**M_2**
$y_n > y_c > y$	**M_3**

2. **In Steep sloped channels ($y_c > y_n$)**

Condition	**Surface Profile**
$y > y_c > y_n$	**S_1**
$Y_c > y > y_n$	**S_2**
$y_c > y_n > y$	**S_3**

3. **In Critical sloped channels ($y_n = y_c$)**

Condition	**Surface Profile**
$y > y_n = y_c$	**C_1**
$y_n = y_c > y$	**C_3**

4. **In Horizontal Channels (y_n = infinity)**

Condition	**Surface Profile**
$y > y_c$	**H_2**
$y_c > y$	**H_3**

5. **In Adverse sloped channels**

Condition	**Surface Profile**
$y > y_c$	**A_2**
$y_c > y$	**A_3**

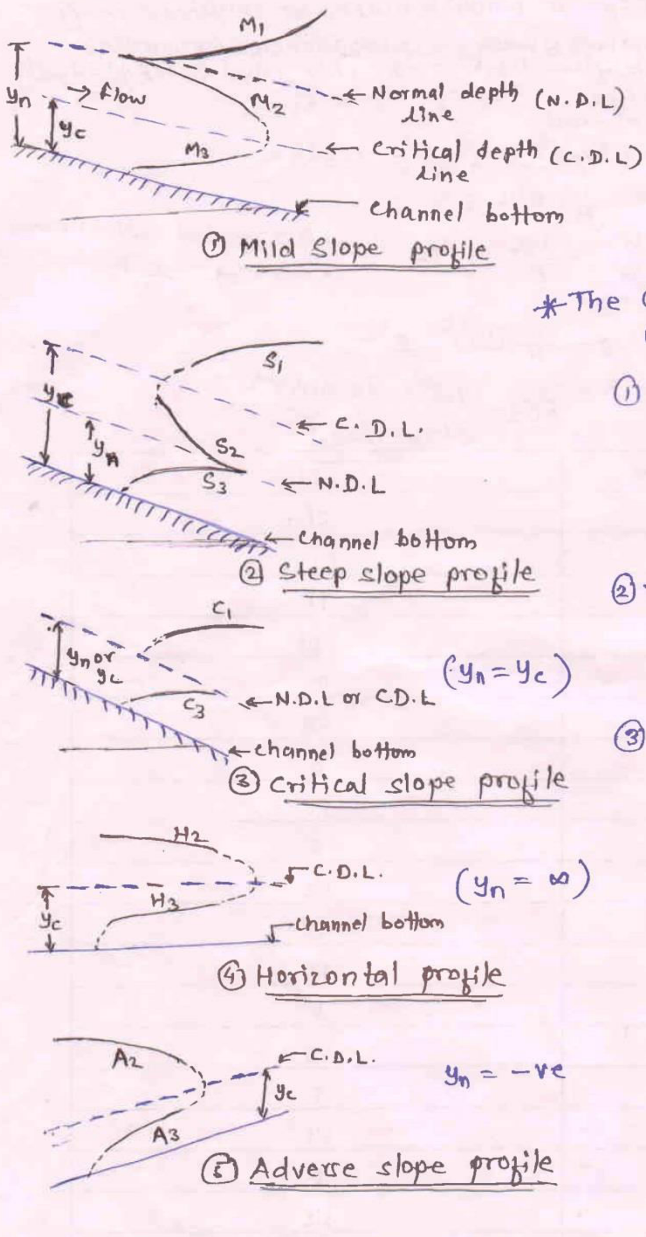


fig. flow profiles of GVF

*The General characteristics of various water surface profiles are:-

1. The water surface profiles with subscript 1 & 3 i.e. $M_1, M_3, S_1, S_3, C_1, C_3, H_3$ & A_3 are Backwater or Rising Curves
2. The water surface profiles with subscript 2 i.e M_2, S_2, H_2 & A_2 are Drawdown or falling curves.
3. The water surface profile meets the critical depth line normally & the normal depth line asymptotically.

S.No.	RGPV questions	Year	Marks
Q.1	Define the term backwater curve & afflux.	Dec 2014	4

Unit-03/Lecture-03

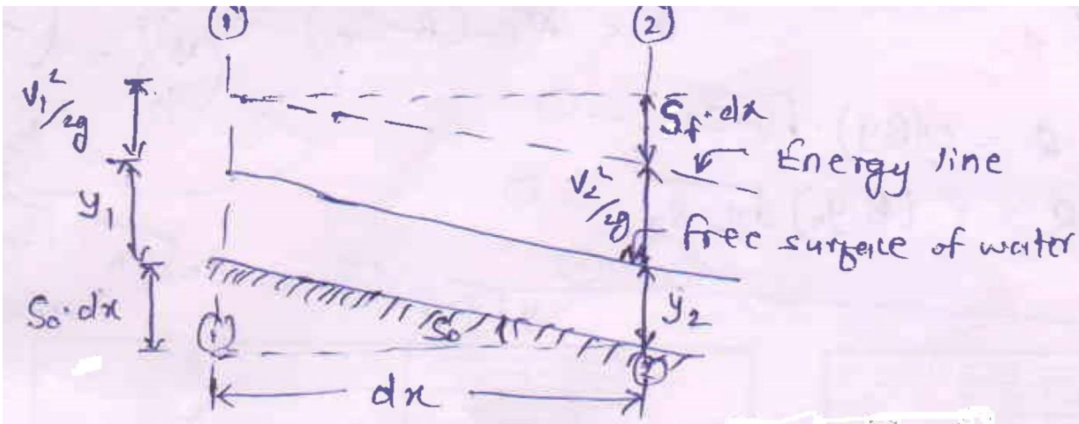
Determination of length of surface profile between two sections

1. Step Method
2. Graphical method
3. Direct integration method

1. Step method

It is the simple method which is applicable for prismatic channels.

In this method, the entire length of the channel is divided into short reaches & the computation is carried out step by step from one end to the other.



Applying Bernoulli's eqn b/w 1-1 & 2-2,

$$S_o\,dx + y_1 + v_1^2/2g = y_2 + v_2^2/2g + h_L$$

Where h_L = energy loss due to friction = S_f dx. Hence

$$S_o\,dx + y_1 + v_1^2/2g = y_2 + v_2^2/2g + S_f\,dx$$

Or

$$E_1 + S_o\,dx = E_2 + S_f\,dx$$

Or

$$\boldsymbol{dx = \frac{E_2 - E_1}{S_o - S_f}}$$

Note: The above expression can also be used for calculating the **length of backwater curve** (L), where dx can be replaced by L. The value of **S_f (slope of energy line)** is calculated either by **Manning's or Chezy's formula**.

2. Graphical method

It is more accurate method than the previous. It is applicable for both prismatic & non-prismatic channels. But it is complicated for large length of surface profiles.

Dynamic eqn of GVF, $\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{v^2}{g\,y}}$

Let f(y) = $\frac{dy}{dx} = \frac{1-\frac{v^2}{g\,y}}{S_o - S_f}$, where v^2/gy & S_f are functions of y. Hence

$$dx = f(y)\,dy$$

Integrating between control sections 1-1 & 2-2

$$\int_{x_1}^{x_2} dx = \int_{y_1}^{y_2} f(y)\,dy$$

A curve is plotted between y & $f(y)$ values. For different values of y, corresponding $f(y)$ values can be computed.

3. Direct Integration Method (Bresse's method)

It is applicable for very wide rectangular channels (b > > y). In this method, Chezy's formula is used for the evaluation of the effect of frictional resistance to the flow. The general varied flow equation for this case is

$$\frac{dy}{dx} = \frac{S_o\left\{1 - (\frac{y_n}{y})^3\right\}}{\left\{1 - (\frac{y_c}{y})^3\right\}}$$

Now if $(y/y_n) = u$, then $dy = y_n\,du$. Putting in above expression & then integrating, we get the Bresse's solution as

$$x = \frac{y_n}{S_0}\left[q - \left\{1 - \left(\frac{y_c^3}{y_n^3}\right)\right\}\int \frac{dq}{1 - q^3}\right] + A$$

Where $\int \frac{dq}{1-q^3}$ = Bresse's varied flow function & A is the constant of integration.

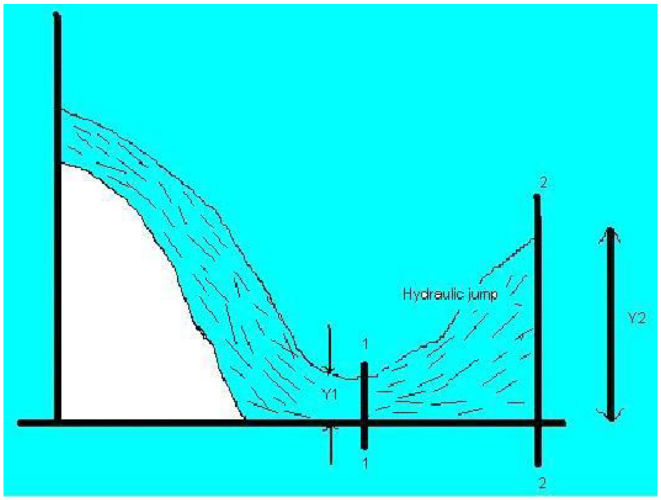
These solutions are very complicated.

S.No.	RGPV questions	Year	Marks
Q.1	Describe various methods of computation of length of flow profiles. State their relative merits & demerits.	Dec 2011	10

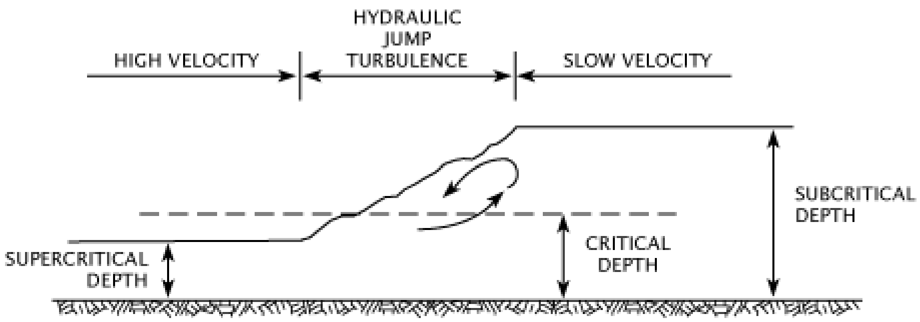
Unit-03/Lecture-04

Rapidly varied flow (RVF) & Hydraulic jump (or Standing waves)

If the depth of flow in a channel changes abruptly over a small length of the channel, the flow is said to be RVF. In RVF conditions, standing waves or eddies are formed which causes loss of energy & the flow changes from supercritical stage to subcritical stage, it is called stage of "Hydraulic jump" in which rise of water takes place.



it can be defined as rise in the level of water when the water transforms its super critical form into the sub critical one i.e. from unstable state to the stable one. This phenomenon can be observed on spillways of the dam. As it's the nature of water to gain its stability after it is being unstable. The place where hydraulic jump occurs, a lot of energy is dissipated in the form of heat energy. We can say that hydraulic jump is dissipator of extra energy of water. This phenomenon is usually used in spillways of the dam to reduce the erosive power of super critical flow. Otherwise it will affect the dam structure.



Effect of Hydraulic Jump

It is very common in the field of hydraulics to use hydraulic jump. It is used to perform different functions. Some of the effects of the hydraulic jump are as under:

- Actually the hydraulic jump usually acts as the energy dissipator. It clears the surplus energy of water.
- Due to the hydraulic jump, many noticeable able disturbances are created in the flowing water like eddies, reverse flow.

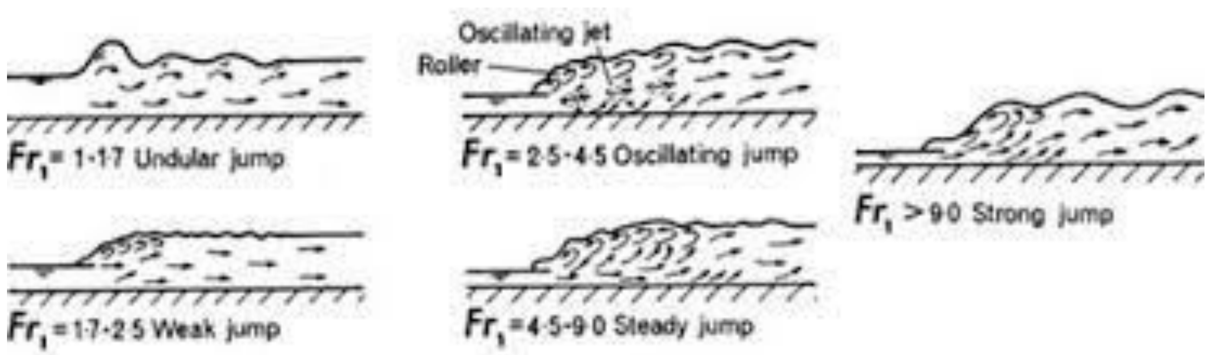
- Usually when the hydraulic jump takes place, the considerable amount of air is trapped in the water. That air can be helpful in removing the wastes in the streams that are causing pollution.
- Hydraulic jump also make the work of different hydraulic structures, effective like weirs, notches and flumes etc.

Applications of Hydraulic Jump:

1. Usually hydraulic jump reverses the flow of water. This phenomenon can be used to mix chemicals for water purification.
2. Hydraulic jump usually maintains the high water level on the downstream side. This high water level can be used for irrigation purposes.
3. Hydraulic jump can be used to remove the air from water supply and sewage lines to prevent the air locking.
4. It prevents the scouring action on the downstream side of the dam structure.

Types of hydraulic jump:

The hydraulic jump is a phenomenon that occurs where there is an abrupt transition from supercritical to subcritical stage of flow. The most important factor that affects the hydraulic jump is the initial Froude number (Fr_1).



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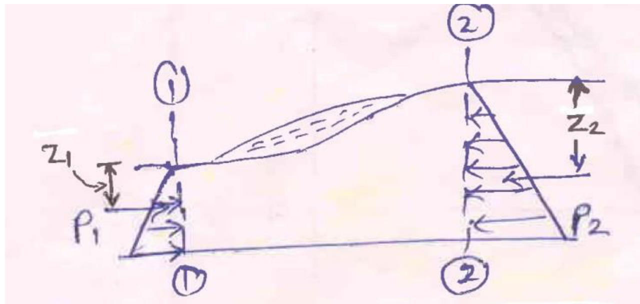
S.No.	RGPV questions	Year	Marks
Q.1	What do you mean by "Standing wave"?	Dec 2014	2
Q.2	Define the following: (i) Weak jump, (ii) Backwater curve, (iii) Drawdown curve, (iv) Undular jump	June 2012	4
Q.3	Define Hydraulic jump.	Dec 2012	2

Unit-03/Lecture-05

Expression for Hydraulic jump:



Assumptions:

1. The flow is uniform & pressure distribution is hydrostatic before & after jump.
2. Frictional losses are neglected.

Derivation:

Let y_1, y_2 = depth of flow at sections 1-1 &2-2 respectively

v_1, v_2 = velocity of flow at the two sections, z_1, z_2 = depth of centroid of cross sectional area from the free surface for the two sections, A_1, A_2 = area of cross section at the sections and P_1 & P_2 = pressure force at the sections.

Force acting on the mass of water b/w 1-1 & 2-2 = Rate of change of momentum in the flow direction

$$P_2 - P_1 = \rho Q (v_1 - v_2)$$

$$P_1 + \rho Q v_1 + = P_2 + \rho Q v_2$$

Putting $v_1 = Q/A_1$, $v_2 = Q/A_2$, $P_1 = \rho g A_1 z_1$, $P_2 = \rho g A_2 z_2$ and dividing by ρg, we have

$$A_1 z_1 + (Q^2 / A_1 g) = A_1 z_1 + (Q^2 / A_2 g)$$

or

$$\mathbf{Az + (Q^2 / Ag) = Constant, F = Specific\ force}$$

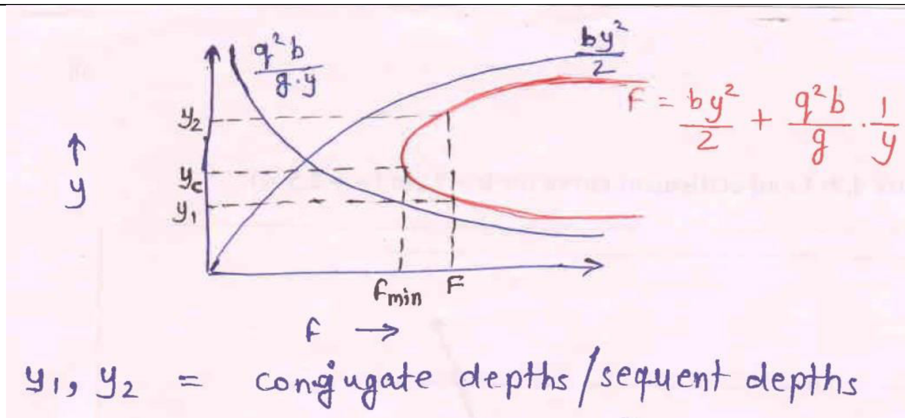
- In RVF, specific force remains constant before & after the jump. It is actually force per unit weight showing the non-uniform flow in open channels.
- For rectangular channels, $A = by$ & $z = y/2$. Hence specific force is given by

  $F = (by)(y/2) + (Q^2 / byg)$

  But discharge per unit width, $q = Q/b$, hence above expression becomes

  $$\mathbf{F = (by^2/2) + (q^2 b/g)(1/y)}$$

  Hence, specific force curve for rectangular channel is given by



- In RVF, at minimum specific force, there is only one depth of flow i.e. critical depth (y_c). At any other greater value of specific force, there will be two depths of flow, called conjugate depths (y_1 and y_2).

  y_1 = pre-jumped depth (or depth before hydraulic jump),

  y_2 = post-jumped depth (or depth after hydraulic jump).

UNIT 3/LECTURE 6

Relation between y_1 & y_2

For rectangular channels,

$$(by_1^2/2) + (q^2 b/g)(1/y_1) = (by_2^2/2) + (q^2 b/g)(1/y_2)$$

Or

$$2q^2/g = (y_1 + y_2)\, y_1 y_2$$

Or

$$y_2 = \frac{y_1}{2}\left\{-1 \pm \sqrt{1 + \frac{8\,q^2}{g\,y_1^3}}\right\}$$

since negative depth is not possible, hence

$$y_2 = \frac{y_1}{2}\left\{-1 + \sqrt{1 + \frac{8\,q^2}{g\,y_1^3}}\right\}$$

now, Froude no. on the upstream side of the jump , $F_e = \frac{v}{\sqrt{gy}} = \frac{q}{\sqrt{gy^3}}$

hence

$$\mathbf{y_2 = \frac{y_1}{2}\left\{-1 + \sqrt{1 + 8\,F_e^2}\right\}}$$

Sequent Depth Ratio: the ratio y_2/y_1 in the above expression can be expressed as sequent depth ratio.

Depth of hydraulic jump: $y_2 - y_1$

Loss of energy due to hydraulic jump (or hydraulic temperature):

When hydraulic jump takes place, loss of energy occurs due to formation of eddies. This loss of energy is equal to the difference in specific energies b/w sections 1-1 & 2-2. Since $E_1 > E_2$, Hence

$$h_L = E_1 - E_2$$

$$h_L = \{y_1 + q^2/2gy_1^2\} - \{y_2 + q^2/2gy_2^2\}$$

$$h_L = q^2/2g\,\{(1/y_1^2) - (1/y_2^2)\}$$

But

$$2q^2/g = (y_1 + y_2)\, y_1 y_2$$

Hence on solving, above expression becomes

$$\boldsymbol{h_L = \frac{(y_2 - y_1)^3}{4 y_1\, y_2}}$$

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	What do you mean by a hydraulic jump? Obtain an expression for depth after the hydraulic jump.	Dec 2013	7
Q.2	Define the sequent depth ratio.	June 2012	2
Q.3	Derive an expression for loss of energy for hydraulic temperature.	Dec 2009	10

UNIT 3/LECTURE 7

Venturi flume

It is a structure in a channel which has a contracted section called throat, d/s of which follows a flared transition section designed to restore the stream to its original width. It is an open channel counterpart of a venturi meter , which is used for measuring discharge in an open channel. A venturi meter would normally measure in millimetres, whereas a venturi flume measures in metres.

At the throat section there will be a drop in water surface & it may be related to the discharge. The velocity of flow at the throat is always less than the critical velocity & hence the discharge passing through it will be a function of the difference between the depths of flow u/s of the entrance section & at the throat. Further since the velocity of flow at the throat is less than the critical velocity, standing wave or hydraulic jump will not occur at any section of the venturiflume.

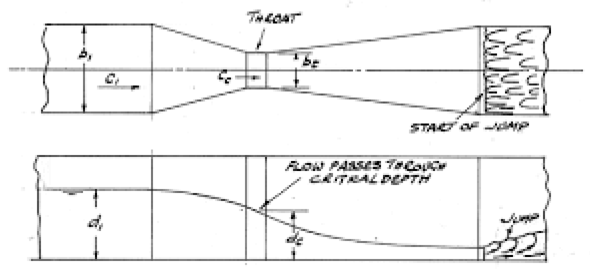
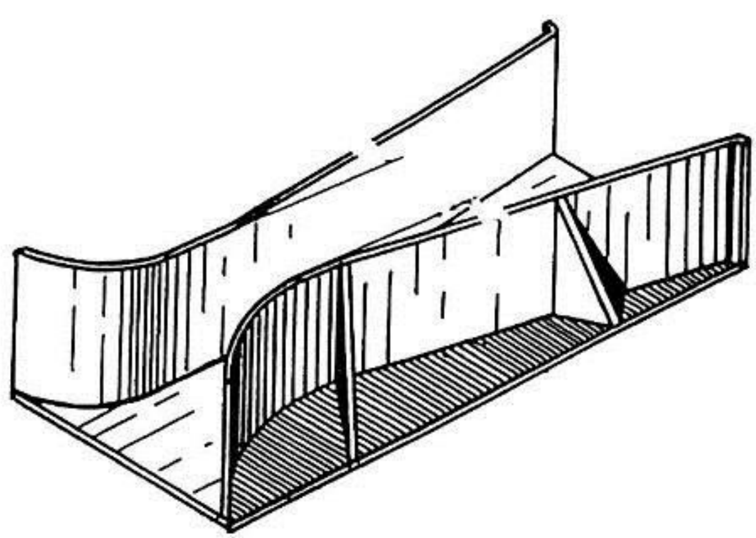


Figure 10-26

Measurement of discharge with venturi flumes requires two measurements, one upstream and one at the throat (narrowest cross-section), if the flow passes in a subcritical state through the flume. If the flumes are designed so as to pass the flow from sub critical to supercritical state while passing through the flume, a single measurement at the throat (which in this case becomes a critical section) is sufficient for computation of discharge. To ensure the occurrence of critical depth at the throat, the flumes are usually designed in such way as to form a hydraulic jump on the downstream side of the structure. These flumes are called 'standing wave flumes’.



For constant specific energy conditions b/w two sections 1-1 & 2-2,

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

Or

$$y_1 - y_2 = \frac{V_2^2}{2g}\left[1 - \frac{V_1^2}{V_2^2}\right] = \frac{V_2^2}{2g}\left[1 - \frac{A_2^2}{A_1^2}\right]$$

Or

$$V_2 = \frac{A_1\sqrt{2g(y_1 - y_2)}}{\sqrt{A_1^2 - A_2^2}}$$

Hence, Rate of flow

$$\mathbf{Q} = A_2 V_2 = \frac{A_1 A_2\sqrt{2g(y_1 - y_2)}}{\sqrt{A_1^2 - A_2^2}}$$

Introducing a coefficient of discharge C_d for the venture flume,

$$\mathbf{Q} = \frac{C_d A_1 A_2\sqrt{2g(y_1 - y_2)}}{\sqrt{A_1^2 - A_2^2}}$$

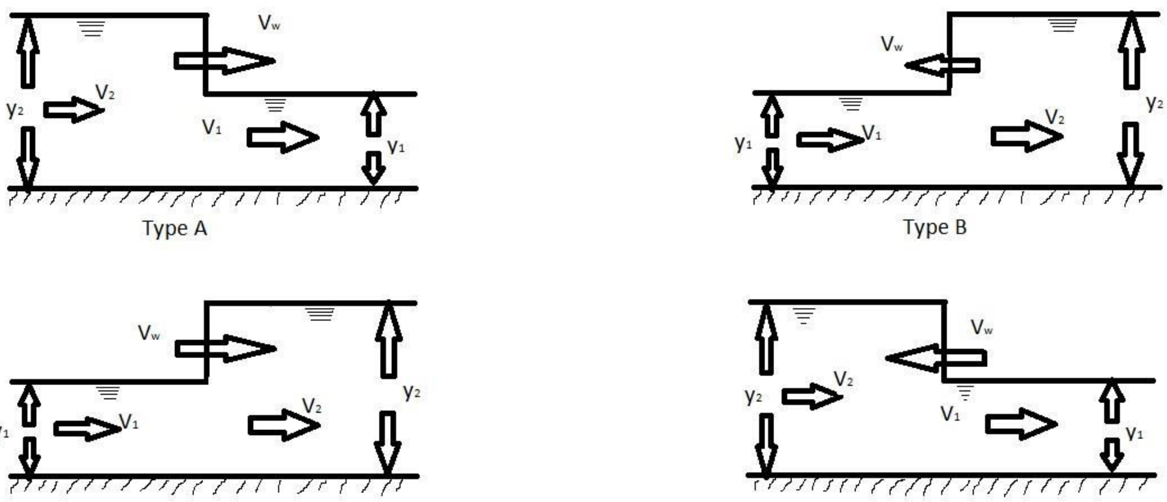
C_d depends upon smoothness of the bed surface & sides, and on the roundness of the corners.

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Derive an expression for the discharge through a venturiflume.	Dec 2014	7
Q.2	Explain in detail, the venturiflumes.	Dec 2009, Dec 2010, June 2015	5, 5, 3

UNIT 3/ LECTURE 8

Surges in open channel

A surge or surge wave is a moving wave front which brings about an abrupt change in depth of flow. It is also referred as moving hydraulic jump & is caused by sudden increase or decrease of flow, which occurs due to sudden opening or closing of a gate fixed in the channel.



Types of surges:

1. **Positive surge**: It results in an increase in the depth of flow. Fig A & B shows two types of positive surges. Type A is a positive surge having an advancing wave front moving downstream. Type B is a positive surge having an advancing wave front moving upstream.

   The positive surge of type A may occur when a gate provided at the head of the channel is suddenly opened. And type B may occur when a gate provided at the tail end of the channel is suddenly closed.

2. **Negative surge**: It results in a decrease in the depth of flow. Fig C & D shows two types of negative surges. Type C is a negative surge having a retreating wave front moving downstream. Type D is a negative surge having a retreating wave front moving upstream.

The negative surge of type C may occur when a gate provided at the head of the channel is suddenly closed & type D may occur when a gate provided at the tail end of the channel is suddenly opened.

S.NO	RGPV QUESTIONS	Year	Marks
Q.1	Explain the surges in open channel.	Dec 2014, Dec 2012, Dec 2010, June 2009, Dec 2009	3, 2, 5, 3, 5