UNIT-II

Relational algebra is a procedural query language, which takes instances of relations as input and yields instances of relations as output. It uses operators to perform queries. An operator can be either unary or binary. They accept relations as their input and yield relations as their output. Relational algebra is performed recursively on a relation and intermediate results are also considered relations.

In the formal relational model terminology, a row is called a tuple, a column header is called an attribute, and the table is called a relation. The data type describing the types of values that can appear in each column is represented by a domain of possible values. We now define these terms—domain, tuple, attribute, and relation formally.

Domains, Attributes, Tuples, and Relations

<table>
<thead>
<tr>
<th>Deposit Relation</th>
<th>Customer Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>bname</strong></td>
<td><strong>Name</strong></td>
</tr>
<tr>
<td><strong>Account</strong></td>
<td><strong>SBI1200</strong></td>
</tr>
<tr>
<td><strong>Enname</strong></td>
<td><strong>Ramesh</strong></td>
</tr>
<tr>
<td><strong>Balance</strong></td>
<td><strong>Jhon</strong></td>
</tr>
</tbody>
</table>

- It has four attributes.
- For each attribute there is a permitted set of values, called the domain of that attribute.
- E.g. the domain of bname is the set of all branch names.

Let $D_1$ denote the domain of bname, and $D_2, D_3$ and $D_4$ the remaining attributes' domains respectively.

A domain D is a set of atomic values. By atomic we mean that each value in the domain is indivisible as far as the formal relational model is concerned. A common method of specifying a domain is to specify a data type from which the data values forming the domain are drawn. It is also useful to specify a name for the domain, to help in interpreting its values. Some examples of domains follow:

- Usa_phone_numbers. The set of ten-digit phone numbers valid in the United States.
- Local_phone_numbers. The set of seven-digit phone numbers valid within a particular area code in the United States. The use of local phone numbers is quickly becoming obsolete, being replaced by standard ten-digit numbers.

A relation schema $R$, denoted by $R(A_1, A_2, ..., A_n)$, is made up of a relation name $R$ and a list of attributes, $A_1, A_2, ..., A_n$. Each attribute $A_i$ is the name of a role played by some domain $D$ in the relation schema $R$. $D$ is called the domain of $A_i$ and is denoted by dom $(A_i)$. A relation schema is used to describe a relation $R$ is called the name of this relation. The degree (or arity) of a relation is the number of attributes $n$ of its relation schema.

Using the datatype of each attribute, the definition is sometimes written as:

STUDENT (Name: string, Ssn: string, Homophone: string, Address: string, Office phone: string, Age: integer, Gpa: real)

Characteristics of Relations

Ordering of Tuples in a Relation. A relation is defined as a set of tuples. Mathematically, elements of a set have no order among them hence, tuples in a relation do not have any particular order.

Ordering of Values within a Tuple and an Alternative Definition of a Relation. According to the preceding definition of a relation, an n-tuple is an ordered list of n values, so the ordering of values in a tuple and hence of attributes in a relation schema is important.

Values and NULLs in the Tuples. Each value in a tuple is an atomic value that is, it is not divisible into
components within the framework of the basic relational model. Hence, composite and multivalued attributes are not allowed. An important concept is that of NULL values, which are used to represent the values of attributes that may be unknown or may not apply to a tuple.

**Relational Model Notation**

A relation schema \( R \) of degree \( n \) is denoted by \( R (A_1, A_2, ..., A_n) \).

- The uppercase letters \( Q, R, S \) denote relation names.
- The lowercase letters \( q, r, s \) denotes relation states.
- The letters \( t, u, v \) denotes tuples.
- In general, the name of a relation schema such as \( \text{STUDENT} \) also indicates the current set of tuples in that relation—the current relation state—whereas \( \text{STUDENT} (\text{Name}, \text{Ssn}, ...) \) refers only to the relation schema.
- An attribute \( A \) can be qualified with the relation name \( R \) to which it belongs by using the dot notation \( R.A \)—for example, \( \text{STUDENT}. \text{Name} \) or \( \text{STUDENT}. \text{Age} \). This is because the same name may be used for two attributes in different relations.

**Relational Model Constraints and Relational Database Schemas**

Constraints on databases can generally be divided into three main categories:

1. Constraints that are inherent in the data model. We call these inherent model-based constraints or implicit constraints. Example: In the relational model, no two tuples in a relation can be duplicates. Why?

2. Constraints that can be directly expressed in schemas of the data model, typically by specifying them in the DDL. We call these schema-based constraints or explicit constraints.

3. Constraints that cannot be directly expressed in the schemas of the data model, and hence must be expressed and enforced by the application programs. We call these application-based or semantic constraints or business rules.

**Types of Schema based Constraints**

**Domain Constraints**

Domain constraints specify that within each tuple, the value of each attribute \( A \) must be an atomic value from the domain \( \text{dom}(A) \).

**Key Constraints and Constraints on NULL Values**

In the formal relational model, a relation is defined as a set of tuples. By definition, all elements of a set are distinct; hence, all tuples in a relation must also be distinct.

A super key \( \text{SK} \) specifies a uniqueness constraint that no two distinct tuples in any state \( r \) of \( R \) can have the same value for \( \text{SK} \). Every relation has at least one default super key—the set of all its attributes. A super key can have redundant attributes, however, so a more useful concept is that of a key, which has no redundancy.

A relation schema may have more than one key. In this case, each of the keys is called a **candidate key**. For example, the \( \text{CAR} \) relation has \( (\text{Licence_no}, \text{Eng_sr_no}, \text{Model}, \text{Make_year}) \) two candidate keys: License_number and Engine_serial_number. It is common to designate one of the candidate keys as the **primary key** of the relation. This is the candidate key whose values are used to identify tuples in the relation.

**Relational Databases and Relational Database Schemas**

The definitions and constraints we have discussed so far apply to single relations and their attributes. A relational database usually contains many relations, with tuples in relations that are related in various ways.
In this section, we define a relational database and a relational database schema. A relational database schema $S$ is a set of relation schemas $S = \{ R_1, R_2, \ldots, R_m \}$ and a set of integrity constraints $IC$. A relational database state $10$ $DB$ of $S$ is a set of relation states $DB = \{ r_1, r_2, \ldots, r_m \}$ such that each $r_i$ is a state of $R_i$ and such that the $r_i$ relation states satisfy the integrity constraints specified in $IC$. Below Figure shows a relational database schema that we call $COMPANY = \{EMPLOYEE, DEPARTMENT, DEPT_LOCATIONS, PROJECT, WORKS_ON, DEPENDENT \}$. The underlined attributes represent primary keys.

**Integrity, Referential Integrity, and Foreign Keys**

The entity integrity constraint states that no primary key value can be NULL. This is because the primary key value is used to identify individual tuples in a relation. Having NULL values for the primary key implies that we cannot identify some tuples. For example, if two or more tuples had NULL for their primary keys, we may not be able to distinguish them if we try to reference them from other relations.

The referential integrity constraint is specified between two relations and is used to maintain the consistency among tuples in the two relations. Informally, the referential integrity constraint states that a tuple in one relation that refers to another relation must refer to an existing tuple in that relation.
The relational algebra is very important for several reasons. First, it provides a formal foundation for relational model operations. Second, and perhaps more important, it is used as a basis for implementing and optimizing queries in the query processing and optimization modules that are integral parts of relational database management systems (RDBMSs).

**Difference Between Calculus & algebra**

The algebra defines a set of operations for the relational model, the relational calculus provides a higher-level declarative language for specifying relational queries. A relational calculus expression creates a new relation. In a relational calculus expression, there is no order of operations to specify how to retrieve the query result only what information the result should contain.

The fundamental operations of relational algebra are as follows –
- Select
- Project
- Union
- Set different
- Cartesian product
- Rename

We will discuss all these operations in the following sections.

**Select Operation (σ)**

It selects tuples that satisfy the given predicate from a relation.

**Notation – σ(r)**

Where σ stands for selection predicate and r stands for relation. p is propositional logic formula which may use connectors like and, or, and not. These terms may use relational operators like =, ≠, ≥, ≤, <, >.

For example –

σsubject = "database"(Books)
Output – Selects tuples from books where subject is 'database'.

σsubject = "database" and price = "450"(Books)
Output – Selects tuples from books where subject is 'database' and 'price' is 450.

σsubject = "database" and price = "450" or year > "2010"(Books)
Output – Selects tuples from books where subject is 'database' and 'price' is 450 or those books published after 2010.

**Project Operation (∏)**

It projects column(s) that satisfy a given predicate.

**Notation – ∏A1, A2, An (r)**

Where A1, A2 , An are attribute names of relation r.
Duplicate rows are automatically eliminated, as relation is a set.

For example –

∏subject, author (Books)
Selects and projects columns named as subject and author from the relation Books.

**Union Operation (U)**

It performs binary union between two given relations and is defined as –
\[ r \cup s = \{ t \mid t \in r \text{ or } t \in s \} \]

Notation - \( r \cup s \)

Where \( r \) and \( s \) are either database relations or relation result set (temporary relation).
For a union operation to be valid, the following conditions must hold –

- \( r \) and \( s \) must have the same number of attributes.
- Attribute domains must be compatible.
- Duplicate tuples are automatically eliminated.
- \( \forall \text{author (Books)} \cup \forall \text{author (Articles)} \)

Output – Projects the names of the authors who have either written a book or an article or both.

**Set Difference (\(-\))**
The result of set difference operation is tuples, which are present in one relation but are not in the second relation.

Notation - \( r - s \)
Finds all the tuples that are present in \( r \) but not in \( s \).

\( \forall \text{author (Books)} - \forall \text{author (Articles)} \)
Output – Provides the name of authors who have written books but not articles.

**Cartesian Product (X)**
Combines information of two different relations into one.

Notation - \( r \times s \)
Where \( r \) and \( s \) are relations and their output will be defined as –

\[ r \times s = \{ q \times t \mid q \in r \text{ and } t \in s \} \]

\( \sigma_{\text{author} = 'tt'}(\text{Books} \times \text{Articles}) \)
Output – Yields a relation, which shows all the books and articles written by tt.

**Rename Operation (\(\rho\))**
The results of relational algebra are also relations but without any name. The rename operation allows us to rename the output relation. 'rename' operation is denoted with small Greek letter rho \( \rho \).

Notation - \( \rho x (E) \)

Where the result of expression \( E \) is saved with name of \( x \).

Additional operations are –
- Set intersection
- Assignment
- Natural join

We can define the three operations UNION, INTERSECTION, and SET DIFFERENCE on two union-compatible relations \( R \) and \( S \) as follows:

- **UNION**: The result of this operation, denoted by \( R \cup S \), is a relation that includes all tuples that are either in \( R \) or in \( S \) or in both \( R \) and \( S \). Duplicate tuples are eliminated.
- **INTERSECTION**: The result of this operation, denoted by \( R \cap S \), is a relation that includes all tuples that are in both \( R \) and \( S \).
- **SET DIFFERENCE (or MINUS)**: The result of this operation, denoted by \( R - S \), is a relation that includes all tuples that are in \( R \) but not in \( S \).

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations.
(b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT − INSTRUCTOR. 
(e) INSTRUCTOR − STUDENT.

CARTESIAN PRODUCT operation—also known as CROSS PRODUCT or CROSS JOIN—which is denoted by \( \times \).
This is also a binary set operation, but the relations on which it is applied do not have to be union compatible.
The JOIN operation, denoted by, is used to combine related tuples from two relations into single “longer” tuples. This operation is very important for any relational database with more than a single relation because it allows us to process relationships among relations.

\[
\text{DEPT}_\text{MGR} \leftarrow \text{DEPARTMENT} \times_{\text{Mgr_ssn=Ssn}} \text{EMPLOYEE} \\
\text{RESULT} \leftarrow \Pi_{\text{Name}, \text{Lname}} (\text{DEPT}_\text{MGR})
\]

Variations of JOIN: The EQUIJOIN and NATURAL JOIN
The most common use of JOIN involves join conditions with equality comparisons only. Such a JOIN, where the only comparison operator used is =, is called an EQUIJOIN.

The standard definition of NATURAL JOIN requires that the two join attributes (or each pair of join attributes) have the same name in both relations. If this is not the case, a renaming operation is applied first.

\[
\text{PROJ}_\text{DEP} \text{ DEPT}_\text{LOCATIONS} \leftarrow \text{DEPARTMENT} \times \text{DEPT}_\text{LOCATIONS}.
\]

The DIVISION Operation
The DIVISION operation, denoted by ÷, is useful for a special kind of query that sometimes occurs in database applications. An example is Retrieve the names of employees who work on all the projects that ‘John Smith’ works on.

\[
\text{SMITH} \leftarrow \sigma_{\text{Fname} = 'Jhon'} \text{ AND Lname = 'SMITH'} (\text{EMPLOYEE}) \\
\text{SMITH}_\text{PNOS} \leftarrow \pi_{\text{pno}} (\text{WORKS}_\text{ON} \Join_{\text{Essn = Ssn}} \text{SMITH})
\]
<table>
<thead>
<tr>
<th>OPERATIONAL</th>
<th>PURPOSE</th>
<th>NOTATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>SELECT</td>
<td>Selects all tuples that satisfy the selection condition from a relation R.</td>
<td>$\sigma&lt;$selection condition$&gt;(R)$</td>
</tr>
<tr>
<td>PROJECT</td>
<td>Produces a new relation with only some of the attributes of R, and removes duplicate tuples.</td>
<td>$\pi&lt;$attribute list$&gt;(R)$</td>
</tr>
<tr>
<td>THETA JOIN</td>
<td>Produces all combinations of tuples from R and R1 2 that satisfy the join condition.</td>
<td>R1 &lt;join condition&gt;</td>
</tr>
<tr>
<td>EQUIJOIN</td>
<td>Produces all the combinations of tuples from R1 and R2 that satisfy a join condition with only equality comparisons.</td>
<td>R1 &lt;join condition&gt;</td>
</tr>
<tr>
<td>NATURAL JOIN</td>
<td>Same as EQUIJOIN except that the join attributes of R2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.</td>
<td>R1*&lt;join condition&gt; R2, OR R1* (&lt;join attributes 1&gt;),</td>
</tr>
<tr>
<td>UNION</td>
<td>Produces a relation that includes all the tuples in R1 or R2 or both R1 and R2; R1 and R2 must be union-compatible.</td>
<td>R1 U R2</td>
</tr>
<tr>
<td>INTERSECTION</td>
<td>Produces a relation that includes all the tuples in both R1 and R2; R1 and R2 must be union-compatible.</td>
<td>R1 (\cap) R2</td>
</tr>
<tr>
<td>DIFFERENCE</td>
<td>Produces a relation that includes all the tuples in R1 that are not in R2; R1 and R2 must be union-compatible.</td>
<td>R1 – R2</td>
</tr>
<tr>
<td>CARTESIAN PRODUCT</td>
<td>Produces a relation that has the attributes of R1 and R2 and includes as tuples all possible combinations of tuples from R1 and R2.</td>
<td>R1 (\times) R2</td>
</tr>
<tr>
<td>DIVISION</td>
<td>Produces a relation R(X) that includes all tuples t[X] in R1(Z) that appear in R1 in combination with every tuple from R(Y), where (Z = X \cup Y).</td>
<td>R1(Z) (\div) R2(Y)</td>
</tr>
</tbody>
</table>

Example on Relational Algebra
Query 1. Retrieve the name and address of all employees who work for the ‘Research’ department.

```
RESEARCH_DEPT ← σ Dname = 'Research' (DEPARTMENT)
RESEARCH_EMPS ← (RESEARCH_DEPT → Dnumber = Dno EMPLOYEE)
RESULT ← π Fname, Lname, Address (RESEARCH_EMPS)
```

Query 2. For every project located in ‘Stafford’, list the project number, the controlling department number, and the department manager’s last name, address, and birth date.

```
STAFFORD_PROJs ← σ Plocation = 'stafford' (PROJECT)
CONTR_DEPTS ← (STAFFORD_PROJs → Dnum = Dnumber DEPARTMENT)
PROJ_DEPT_MGRS ← (CONTR_DEPTS → Mgr_ssn = Ssn EMPLOYEE)
RESULT ← π Pnumber, Dnum, Lname, Address, Bdate (PROJ_DEPT_MGRS)
```

Query 3. Find the names of employees who work on all the projects controlled by department number 5.

```
DEPT5_PROJS ← p(Pno)(πPnumber(σDnum=5(PROJECT)))
EMP_PROJ ← p(Ssn, Pno)(πEssn, Pno(WORKS_ON))
RESULT_EMP_SSNS ← EMP_PROJ ÷ DEPT5_PROJS
RESULT ← π Lname, Fname (RESULT_EMP_SSNS * EMPLOYEE)
```
Query 4. List the names of all employees with two or more dependents.

Strictly speaking, this query cannot be done in the basic (original) relational algebra. We have to use the AGGREGATE FUNCTION operation with the COUNT aggregate function. We assume that dependents of the same employee have distinct Dependent_name values.

$T_1(Sn, No\_of\_Dependents) \leftarrow \text{Essn} \ \exists \text{COUNT}_\text{Dependent\_name}(\text{DEPENDENT})$

$T_2 \leftarrow \sigma_{\text{No\_of\_Dependents} \geq 2}(T_1)$

RESULT $\leftarrow \pi\text{Name, Fname}(T_2 \bowtie \text{EMPLOYEE})$

<table>
<thead>
<tr>
<th>ID</th>
<th>Name</th>
<th>Dept_name</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>10101</td>
<td>Shrinivasan</td>
<td>Comp.sci</td>
<td>65000</td>
</tr>
<tr>
<td>12121</td>
<td>Wu</td>
<td>Finance</td>
<td>90000</td>
</tr>
<tr>
<td>15151</td>
<td>Mozart</td>
<td>Music</td>
<td>40000</td>
</tr>
<tr>
<td>22222</td>
<td>Einstin</td>
<td>Physics</td>
<td>95000</td>
</tr>
</tbody>
</table>

Instructor Relation

instructor relation where the instructor is in the “Physics” department, we write:

dept name = “Physics” (instructor )

Relational Calculus

In contrast to Relational Algebra, Relational Calculus is a non-procedural query language, that is, it tells what to do but never explains how to do it.

Relational calculus exists in two forms –

Tuple Relational Calculus (TRC) Filtering variable ranges over tuples

Notation – $\{ T | \text{Condition} \}$

Returns all tuples T that satisfies a condition.

For example –

$\{ T.\text{name} \ | \ \text{Author}(T) \ \text{AND} \ T.\text{article} = 'database' \}$

Output – Returns tuples with 'name' from Author who has written article on 'database'.

TRC can be quantified. We can use Existential ($\exists$) and Universal Quantifiers ($\forall$).

For example –

$\{ R | \exists T \ \in \text{Authors}(T.\text{article}='database' \ \text{AND} \ R.\text{name}=T.\text{name}) \}$

Output – The above query will yield the same result as the previous one.

Domain Relational Calculus (DRC)

In DRC, the filtering variable uses the domain of attributes instead of entire tuple values (as done in TRC, mentioned above).

Notation –

$\{ a_1, a_2, a_3, ..., an \ | \ P(a_1, a_2, a_3, ..., an) \}$
Where \( a_1, a_2 \) are attributes and \( P \) stands for formulae built by inner attributes.

For example –

\[
\{<\text{article}, \text{page}, \text{subject}> | \in TP \land \text{subject} = \text{database}'\}
\]

Output – Yields Article, Page, and Subject from the relation TP where subject is database.

Just like TRC, DRC can also be written using existential and universal quantifiers. DRC also involves relational operators.

The expression power of Tuple Relation Calculus and Domain Relational Calculus is equivalent to Relational Algebra.

**E-R Diagram**

An ER schema diagram for the COMPANY database

**Entities and Attributes**

**Entities and Their Attributes.** The basic object that the ER model represents is an **entity**, which is a **thing** in the real world with an independent existence. An entity may be an object with a physical existence (for example, a particular person, car, house, or employee) or it may be an object with a conceptual existence (for instance, a company, a job, or a university course).

**Composite versus Simple (Atomic) Attributes.** **Composite attributes** can be divided into smaller subparts, which represent more basic attributes with independent meanings. For example, the Address attribute of the EMPLOYEE entity.
**Single-Valued versus Multivalued Attributes.** Most attributes have a single value for a particular entity; such attributes are called *single-valued*. For example, Age.

A *multivalued* attribute may have lower and upper bounds to constrain the *number of values* allowed for each individual entity. For example, the Colors attribute of a car.

**Stored versus Derived Attributes.** In some cases, two (or more) attribute values are related—for example, the Age and Birth_date attributes of a person.

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**Initial Conceptual Design of the COMPANY Database covers**

**Relationship & type**

**Cardinality**

**Week Entity**

**Participation Constraints:** The participation constraint specifies whether the existence of an entity depends on its being related to another entity via the relationship type.

Company schema, with structural constraints specified using (min, max) notation and role names.

**Enhanced Entity-Relationship (EER) Model**

Semantic data modeling concepts that were incorporated into conceptual data models such as the ER Model. ER model can be enhanced to include these concepts, leading to the Enhanced ER (EER) model.

**Subclasses:** An entity type is used to represent both a type of entity and the entity set or collection of entities of that type that exist in the database. For example, the entity type EMPLOYEE describes the type (that is, the attributes and relationships) of each employee entity, and also refers to the current set of EMPLOYEE entities in the COMPANY database.

**Super classes:** We call each of these subgroupings a subclass or subtype of the EMPLOYEE entity type, and the EMPLOYEE entity type is called the superclass or super type for each of these subclasses.
Extended E-R Model

Specialization

Specialization is the process of defining a set of subclasses of an entity type. This entity type is called the superclass of the specialization. The set of subclasses that forms a specialization is defined on the basis of some distinguishing characteristic of the entities in the superclass. For example, the set of subclasses {SECRETARY, ENGINEER, TECHNICIAN} is a specialization of the superclass EMPLOYEE that distinguishes among employee entities based on the job type of each employee entity.

Steps for Specialization

- Define a set of subclasses of an entity type.
- Establish additional specific attributes with each subclass.
- Establish additional specific relationship types between each subclass and other entity types or other subclasses.

EER diagram notation to represent subclasses and specialization. Three specializations of EMPLOYEE: {SECRETARY, TECHNICIAN, ENGINEER} {MANAGER} {HOURLY_EMPLOYEE, SALARIED_EMPLOYEE}

Generalization

Generalization

We can think of a reverse process of abstraction in which we suppress the differences among several entity types, identify their common features, and generalize them into a single superclass of which the original entity types are special subclasses. For example, consider the entity types CAR and TRUCK.

Generalization. (a) Two entity types, CAR and TRUCK. (b) Generalizing CAR and TRUCK into the superclass VEHICLE.
Inheritance
We use all the above features of ER-Model in order to create classes of objects in object-oriented programming. The details of entities are generally hidden from the user; this process known as abstraction. Inheritance is an important feature of Generalization and Specialization. It allows lower-level entities to inherit the attributes of higher-level entities.

For example, the attributes of a Person class such as name, age, and gender can be inherited by lower-level entities such as Student or Teacher.

Aggregation
Aggregation is a process when the relation between two entities is treated as a single entity. Here the relation between Center and Course is acting as an Entity in relation with Visitor.