

UNIT-2: INVENTORY CONTROL

INVENTORY

The amount of material, a company has in stock at a specific time is known as inventory or in terms of money it can be defined as the total capital investment over all the materials stocked in the company at any specific time. Inventory may be in the form of,

- raw material inventory
- in process inventory
- finished goods inventory
- spare parts inventory
- Office stationary etc.

As a lot of money is engaged in the inventories along with their high carrying costs, companies cannot afford to have any money tied in excess inventories. Any excessive investment in inventories may prove to be a serious drag on the successful working of an organization. Thus there is a need to manage our inventories more effectively to free the excessive amount of capital engaged in the materials.

Why Inventories?

Inventories are needed because demand and supply cannot be matched for physical and economic reasons. There are several other reasons for carrying inventories in any organization.

- To safe guard against the uncertainties in price fluctuations, supply conditions, demand conditions, lead times, transport contingencies etc.
- To reduce machine idle times by providing enough in-process inventories at appropriate locations.
- To take advantages of quantity discounts, economy of scale in transportation etc.
- To decouple operations i.e. to make one operation's supply independent of another's supply. This helps in minimizing the impact of break downs, shortages etc. on the performance of the downstream operations. Moreover operations can be scheduled independent of each other if operations are decoupled.
- To reduce the material handling cost of semi-finished products by moving them in large quantities between operations.
- To reduce clerical cost associated with order preparation, order procurement etc.

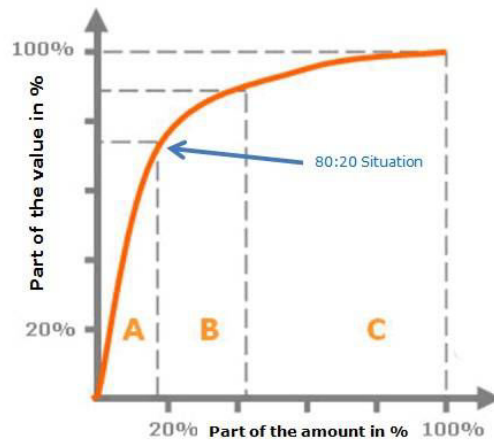
ABC Analysis

Inventory optimization in supply chain, ABC analysis is an inventory categorization method which consists in dividing items into three categories, A, B and C: A being the most valuable items, C being the least valuable ones. This method aims to draw managers' attention on the critical few (A-items) and not on the trivial many (C-items).

The Pareto principle states that 80% of the overall consumption value is based on only 20% of total items. In other words, demand is not evenly distributed between items: top sellers vastly outperform the rest.

The ABC approach states that, when reviewing inventory, a company should rate items from A to C, basing its ratings on the following rules:

- A-items are goods which annual consumption value is the highest. The top 70-80% of the annual consumption value of the company typically accounts for only 10-20% of total inventory items.
- B-items are the interclass items, with a medium consumption value. Those 15-25% of annual consumption value typically accounts for 30% of total inventory items.
- C-items are, on the contrary, items with the lowest consumption value. The lower 5% of the annual consumption value typically accounts for 50% of total inventory items.



The annual consumption value is calculated with the formula: (Annual demand) x (item cost per unit).

Through this categorization, the supply manager can identify inventory hot spots, and separate them from the rest of the items, especially those that are numerous but not that profitable.

Policies based on ABC analysis leverage the sales imbalance outlined by the Pareto principle. This implies that each item should receive a weighed treatment corresponding to its class:

- A-items should have tight inventory control, more secured storage areas and better sales forecasts. Reorders should be frequent, with weekly or even daily reorder. Avoiding stock-outs on A-items is a priority.
- Reordering C-items is made less frequently. A typically inventory policy for C-items consist of having only 1 unit on hand, and of reordering only when an actual purchase is made. This approach leads to stock-out situation after each purchase which can be an acceptable situation, as the C-items present both low demand and higher risk of excessive inventory costs. For C-items, the question is not so much how many units do we store? But rather do we even keep this item in store?
- B-items benefit from an intermediate status between A and C. An important aspect of class B is the monitoring of potential evolution toward class A or, in the contrary, toward the class C.

Inventory Costs

In order to control inventories appropriately, one has to consider all cost elements that are associated with the inventories. There are four such cost elements, which do affect cost of inventory.

Unit cost: it is usually the purchase price of the item under consideration. If unit cost is related with the purchase quantity, it is called as discount price.

Procurement costs: This includes the cost of order preparation, tender placement, cost of postages, telephone costs, receiving costs, set up cost etc.

Carrying costs: This represents the cost of maintaining inventories in the plant. It includes the cost of insurance, security, warehouse rent, taxes, interest on capital engaged, spoilage, breakage etc.

Stock out costs: This represents the cost of loss of demand due to shortage in supplies. This includes cost of loss of profit, loss of customer, loss of goodwill, penalty etc.

If one year planning horizon is used, the total annual cost of inventory can be expressed as:

Total annual inventory cost = Cost of items + Annual procurement cost + Annual carrying cost + Stock out

cost

Variables in Inventory Models

D = Total annual demand (in units)

Q = Quantity ordered (in units)

Q* = Optimal order quantity (in units)

R = Reorder point (in units)

R* = Optimal reorder point (in units)

L = Lead time

S = Procurement cost (per order)

C = Cost of the individual item (cost per unit)

I = Carrying cost per unit carried (as a percentage of unit cost C)

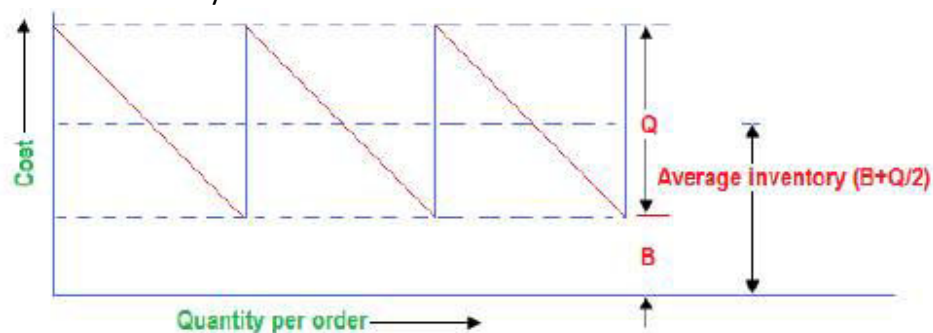
K = Stock out cost per unit out of stock

P = Production rate or delivery rate

dl = Demand per unit time during lead time
DI = Total demand during lead time

TC = Total annual inventory costs

TC* = Minimum total annual inventory costs



Number of orders per year = Annual Demand/Ordered quantity= D/Q

Total procurement cost per year = $S \cdot D / Q$

Total carrying cost per year = Carrying cost per unit * average inventory per cycle
= $I \cdot (B + (Q/2))$

Cost of items per year = Annual demand * unit cost

$$= D.C$$

Total annual inventory cost (TC) = (Ordering Cost) × (Number of order placed in a year) + (Carrying cost per unit) × (Average inventory level during year)

$$TC = D.C + (I.(B+(Q/2))+S.D/Q \quad (\text{When stock out is not considered})$$

The objective of inventory management team is to minimize the total annual inventory cost. A simplified graphical presentation in which cost of items, procurement cost and carrying cost are depicted is shown in below . It can be seen that large values of order quantity Q result in large carrying cost. Similarly, when order quantity Q is large, fewer orders will be placed and procurement cost will decrease accordingly. The total cost curve indicates that the minimum cost point lies at the intersection of carrying cost and procurement cost curves.

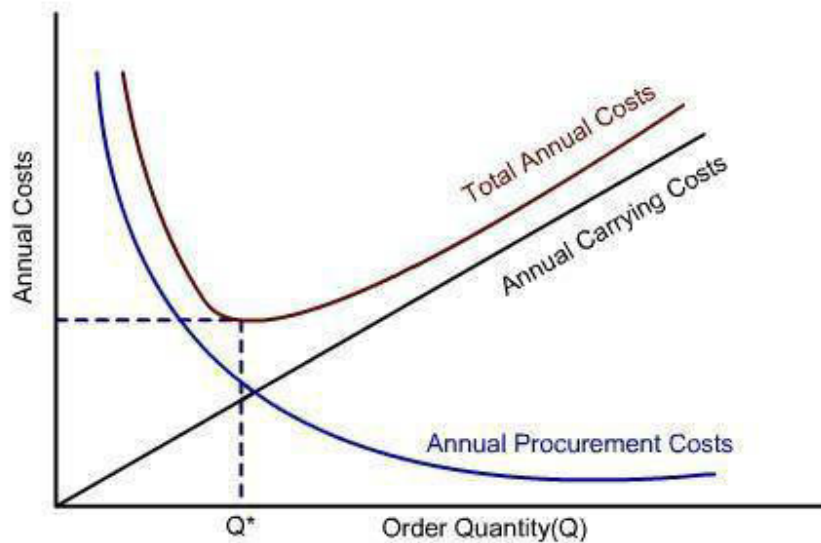


Figure: Inventory Related Costs

Inventory Operating Doctrine

When managing inventories, operations manager has to make two important decisions:

- When to reorder the stock (i.e. time to reorder or reorder point)
- How much stock to reorder (i.e. order quantity)

Reorder point is usually a predetermined inventory level, which signals the operations manager to start the procurement process for the next order. Order quantity is the order size.

Inventory Modelling

Inventory model is a mathematical model that helps business in determining the optimum level of inventories that should be maintained in a production process, managing frequency of ordering, deciding on quantity of goods or raw materials to be stored, tracking flow of supply of raw materials and goods to provide .

Economic Order Quantity (EOQ) Model

This model is applied when objective is to minimize the total annual cost of inventory in the organization. Economic order quantity is that size of the order which helps in attaining the above set objective. EOQ model is applicable under the following conditions.

- Demand per year is deterministic in nature
- Planning period is one year
- Lead time is zero or constant and deterministic in nature
- Replenishment of items is instantaneous
- Demand/consumption rate is uniform and known in advance
- No stockout condition exist in the organization

The total annual cost of the inventory (TC) is given by the following equation in EOQ model.

$$TC = D.C + (I.(B+(Q/2))+S.D/Q \quad (B \text{ is buffer stock})$$

To determine the minimum point of the total cost curve, calculate the derivative of the total cost with respect to Q (assume all other variables are constant) and set it equal to 0

$$d(TC)/d(Q) = 0 + (-SD/Q^2) + I/2 = 0$$

$$Q^2 = 2 * S * D / I$$

$$Q = (2 * S * D / I)^{1/2} \quad (Q \text{ is economic order quantity})$$

Example

- annual requirement quantity (D) = 10000 units
- Cost per order (S) = 40
- Cost per unit (C) = 50
- Yearly Carrying cost percentage (h/P)(percentage of P) = 10%
- Yearly carrying cost per unit (I) = 50 * 10% = 5

$$\text{Economic order quantity} = (2 * 10000 * 40 / 5)^{1/2} = 400 \text{ units}$$

$$\text{Number of orders per year (based on EOQ)} = 10000 / 400 = 25$$

$$\text{Total cost} = 50 * 10000 + 40 * (10000 / 400) + 5 * (400 / 2) = 502000$$

Total cost

If we check the total cost for any order quantity other than 400(=EOQ), we will see that the cost is higher. For instance, supposing 300 units per order, then

This illustrates that the economic order quantity is always in the best interests of the firm.

STATISTICAL QUALITY CONTROL

Statistical quality control (SQC) is the term used to describe the set of statistical tools used by quality professionals, tools that can help us make the right quality decisions.

Descriptive statistics are used to describe quality characteristics and relationships. Included are statistics such as the mean, standard deviation, the range, and a measure of the distribution of data.

Statistical process control (SPC) involves inspecting a random sample of the output from a process and deciding whether the process is producing products with characteristics that fall within a predetermined range. SPC answers the question of whether the process is functioning properly or not.

Acceptance sampling is the process of randomly inspecting a sample of goods and deciding whether to accept the entire lot based on the results. Acceptance sampling determines whether a batch of goods should be accepted or rejected.

No two products are exactly alike because of slight differences in materials, workers, machines, tools, and other factors. These are called **common, or random, causes of variation**. Common causes of variation are based on random causes that we cannot identify. These types of variation are unavoidable and are due to slight differences in processing.

The second type of variation that can be observed involves variations where the causes can be precisely identified and eliminated. These are called **assignable causes of variation**. Examples of this type of variation are poor quality in raw materials, an employee who needs more training, or a machine in need of repair. In each of these examples the problem can be identified and corrected.

DESCRIPTIVE STATISTICS

Mean: A statistic that measures the central tendency of a set of data.

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Where

\bar{x} = the mean

x_i = observation i, i_1, \dots, n

n = number of observations

The Range and Standard Deviation

Range: The difference between the largest and smallest observations in a set of data.

Standard deviation: A statistic that measures the amount of data dispersion around the mean.

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

where

σ = standard deviation of a sample

\bar{x} = the mean

x_i = observation i, i_1, \dots, n

n = the number of observations in the sample

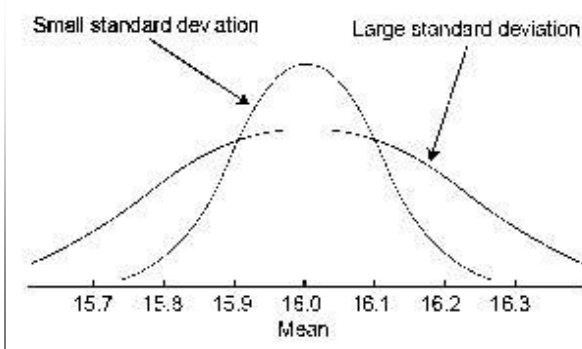


Figure: Normal distributions with varying standard deviations

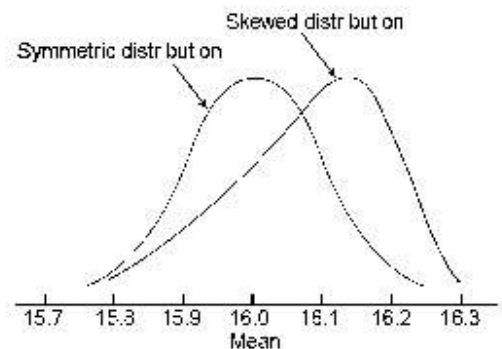
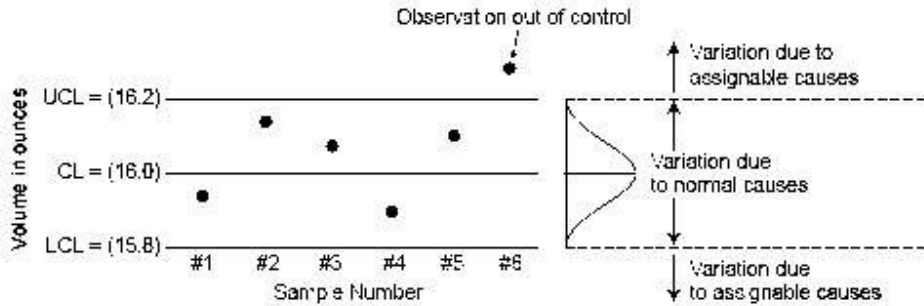


Figure: Differences between symmetric and skewed distributions

Control Charts

A **control chart** (also called process chart or quality control chart) is a graph that shows whether a sample of data falls within the common or normal range of variation. A control chart has upper and lower control limits that separate common from assignable causes of variation. The common range of variation is defined by the use of control chart limits. We say that a process is **out of control** when a plot of data reveals that one or more samples fall outside the control limits.



A sample Quality control chart for the upper control limit (UCL) is the maximum acceptable variation from the mean for a process that is in a state of control. Similarly, the lower control limit (LCL) is the minimum acceptable variation from the mean for a process that is in a state of control.

Types of Control Charts

The different characteristics that can be measured by control charts can be divided into two groups: **variables** and **attributes**.

A *control chart for variables* is used to monitor characteristics that can be measured and have a continuum of values, such as height, weight, or volume.

A *control chart for attributes*, on the other hand, is used to monitor characteristics that have discrete values and can be counted. Often they can be evaluated with a simple yes or no decision. Examples include colour, taste, or smell. The monitoring of attributes usually takes less time than that of variables because a variable needs to be measured

CONTROL CHARTS FOR VARIABLES

Mean (x-Bar) Charts: A control chart used to monitor changes in the mean value of a process.

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_k}{k}$$

To construct the upper and lower control limits of the chart, we use the following formulas:

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

Where

$\bar{\bar{x}}$ = the average of the sample means

z = standard normal variable (2 for 95.44% confidence, 3 for 99.74% confidence)

$\sigma_{\bar{x}}$ = standard deviation of the distribution of sample means, computed as σ/\sqrt{n}

σ = population (process) standard deviation

n = sample size (number of observations per sample)

Example:

Sample Number	Observations (bottle volume in ounces)				Average	Range
	1	2	3	4	\bar{x}	R
1	15.85	16.02	15.83	15.93	15.91	0.19
2	16.12	16.00	15.85	16.01	15.99	0.27
3	16.00	15.91	15.94	15.83	15.92	0.17
4	16.20	15.85	15.74	15.93	15.93	0.46
5	15.74	15.86	16.21	16.10	15.98	0.47
6	15.94	16.01	16.14	16.03	16.03	0.20
7	15.75	16.21	16.01	15.86	15.96	0.46
8	15.82	15.94	16.02	15.94	15.93	0.20
9	16.04	15.98	15.83	15.98	15.96	0.21
10	15.64	15.86	15.94	15.89	15.83	0.30
11	16.11	16.00	16.01	15.82	15.99	0.29
12	15.72	15.85	16.12	16.15	15.96	0.43
13	15.85	15.76	15.74	15.98	15.83	0.24
14	15.73	15.84	15.96	16.10	15.91	0.37
15	16.20	16.01	16.10	15.89	16.05	0.31
16	16.12	16.08	15.83	15.94	15.99	0.29
17	16.01	15.93	15.81	15.68	15.86	0.33
18	15.78	16.04	16.11	16.12	16.01	0.34
19	15.84	15.92	16.05	16.12	15.98	0.28
20	15.92	16.09	16.12	15.93	16.02	0.20
21	16.11	16.02	16.00	15.88	16.00	0.23
22	15.98	15.82	15.89	15.89	15.90	0.16
23	16.05	15.73	15.73	15.93	15.86	0.32
24	16.01	16.01	15.89	15.86	15.94	0.15
25	16.08	15.78	15.92	15.98	15.94	0.30
Total					398.75	7.17

Solution

$\sigma = 0.14$ (Given)

The center line of the control data is the average of the samples:

$$\bar{\bar{x}} = 398.75/25 = 15.95$$

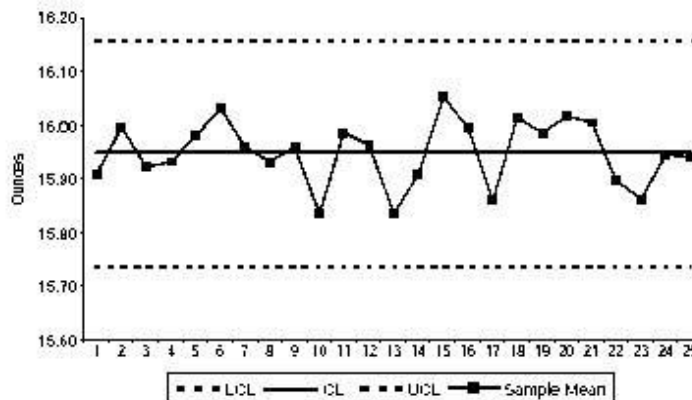
The control limits are

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

$$\text{UCL} = 15.95 + 3(0.14/\sqrt{4}) = 16.16$$

$$\text{LCL} = 15.95 - 3(0.14/\sqrt{4}) = 15.74$$



Range (R) Charts

Range (R) charts are another type of control chart for variables. Whereas x-bar charts measure shift in the central tendency of the process, range charts monitor the dispersion or variability of the process. The method for developing and using R-charts is the same as that for x-bar charts. The center line of the control chart is the average range, and the upper and lower control limits are computed as follows:

$$\begin{aligned} \text{CL} &= \bar{R} \\ \text{UCL} &= D_4 \bar{R} \\ \text{LCL} &= D_3 \bar{R} \end{aligned}$$

Factors for three-sigma control limits of \bar{x} and R-charts (Source: Factors adapted from the *ASTM Manual on Quality Control of Materials*.)

Sample Size n	Factor for \bar{x} -Chart	Factors for R-Chart	
	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59
21	0.17	0.43	1.58
22	0.17	0.43	1.57
23	0.16	0.44	1.56
24	0.16	0.45	1.55
25	0.15	0.46	1.54

The quality control inspector would like to develop a range (R) chart in order to monitor volume dispersion in the bottling process. Use the data from example above to develop control limits for the sample range.

$$\bar{R} = 7.17/25 = 0.29$$

$$n = 4$$

For $n = 4$ from table above, get the value of D_4 and D_3

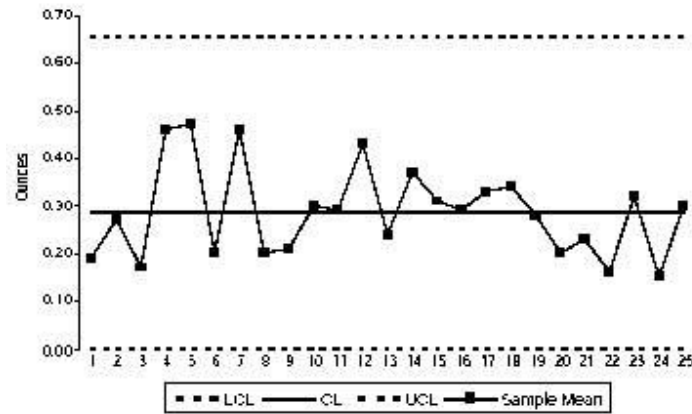
$$D_4 = 2.28, D_3 = 0$$

$$\text{UCL} = 2.28 * 0.29 = 0.6612$$

$$\text{LCL} = 0 * 0.29 = 0$$

$$D_3 = 0$$

The resulting control chart is:



CONTROL CHARTS FOR ATTRIBUTES

Control charts for attributes are used to measure quality characteristics that are counted rather than measured. Attributes are discrete in nature and entail simple yes-or-no decisions. For example, this could be the number of non-functioning light bulbs, the proportion of broken eggs in a carton, the number of rotten apples, the number of scratches on a tile, or the number of complaints issued. Two of the most common types of control charts for attributes are p-charts and c-charts.

P-charts are used to measure the proportion of items in a sample that are defective. Examples are the proportion of broken cookies in a batch and the proportion of cars produced with a misaligned fender. P-charts are appropriate when both the number of defectives measured and the size of the total sample can be counted. A proportion can then be computed and used as the statistic of measurement.

C-charts count the actual number of defects. For example, we can count the number of complaints from customers in a month, the number of bacteria on a petri dish, or the number of barnacles on the bottom of a boat. However, we *cannot* compute the proportion of complaints from customers, the proportion of bacteria on a petri dish, or the proportion of barnacles on the bottom of a boat.

The primary difference between using a p-chart and a c-chart is as follows. A p-chart is used when both the total sample size and the number of defects can be computed. A c-chart is used when we can compute only the number of defects but cannot compute the proportion that is defective.

P-CHARTS

The computation of the center line as well as the upper and lower control limits is similar to the computation for the other kinds of control charts. The center line is computed as the average proportion defective in the population. This is obtained by taking a number of samples of observations at random and computing the average value of 'p' across all samples.

To construct the upper and lower control limits for a p-chart, we use the following formulas:

$$UCL = \bar{p} + z\sigma_p$$

$$LCL = \bar{p} - z\sigma_p$$

Where

z = standard normal variable

\bar{p} = the sample proportion defective

σ_p = the standard deviation of the average proportion defective

The sample standard deviation is computed as follows:

$$\sigma_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

Where n is the sample size

Example

A production manager at a tire manufacturing plant has inspected the number of defective tires in twenty random samples with twenty observations each. Following are the number of defective tires found in each sample:

Sample Number	Number of Defective Tires	Number of Observations Sampled	Fraction Defective
1	3	20	.15
2	2	20	.10
3	1	20	.05
4	2	20	.10
5	1	20	.05
6	3	20	.15
7	3	20	.15
8	2	20	.10
9	1	20	.05
10	2	20	.10
11	3	20	.15
12	2	20	.10
13	2	20	.10
14	1	20	.05
15	1	20	.05
16	2	20	.10
17	4	20	.20
18	3	20	.15
19	1	20	.05
20	1	20	.05
Total	40	400	

Construct a three-sigma control chart (z = 3) with this information.

Solution

The center line of the chart is

$$CL = \bar{p} = \text{total number of defective tires} / \text{total number of observations} = 40 / 400 = 0.1$$

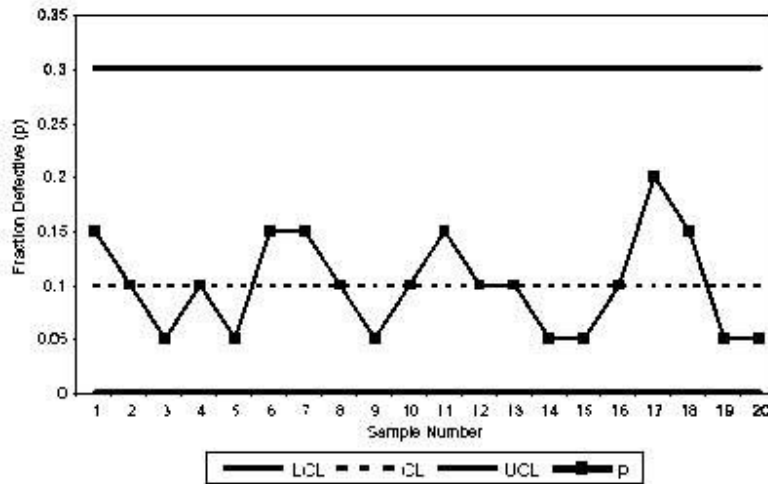
$$\begin{aligned} \sigma_p &= \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \\ &= \sqrt{(0.1 * 0.9 / 20)} = 0.067 \end{aligned}$$

$$UCL = 0.1 + 3 * 0.067 = 0.301$$

$$LCL = 0.1 - 3 * 0.067 = (-) 0.101$$

In this example the lower control limit is negative, which sometimes occurs because the computation is an approximation of the binomial distribution. When this occurs, the LCL is rounded up to zero because we cannot have a negative control limit.

The resulting control chart is as follows:



C-CHARTS

C-charts are used to monitor the number of defects per unit. Examples are the number of returned meals in a restaurant, the number of trucks that exceed their weight limit in a month, the number of discolorations on a square foot of carpet, and the number of bacteria in a milliliter of water. Note that the types of units of measurement we are considering are a period of time, a surface area, or a volume of liquid.

The average number of defects, is the center line of the control chart. The upper and lower control limits are computed as follows:

$$UCL = \bar{c} + z\sqrt{\bar{c}}$$

$$LCL = \bar{c} - z\sqrt{\bar{c}}$$

Example

The number of weekly customer complaints are monitored at a large hotel using a c-chart. Complaints have been recorded over the past twenty weeks. Develop three-sigma control limits using the following data:

																				Total		
Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
No. of																						
Complaints	3	2	3	1	3	3	2	1	3	1	3	4	2	1	1	1	3	2	2	3	44	

Solution

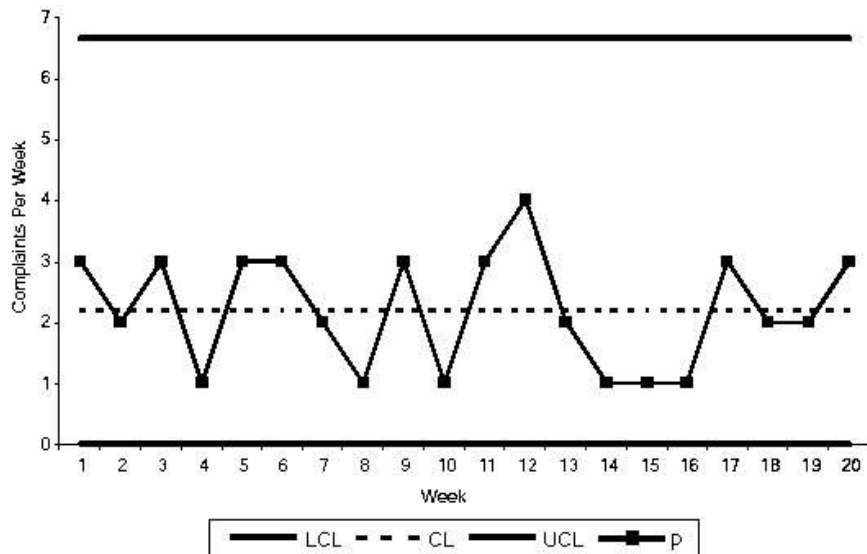
The average number of complaints per week is

$$\bar{c} = 44/20 = 2.2$$

$$UCL = 2.2 + 3*\sqrt{2.2} = 6.65$$

$$LCL = 2.2 - 3*\sqrt{2.2} = -2.25$$

As the LCL is negative and should be rounded up to zero. Following is the control chart for this example:



PROCESS CAPABILITY

Product specifications, often called tolerances, are pre-set ranges of acceptable quality characteristics, such as product dimensions. For a product to be considered acceptable, its characteristics must fall within this pre-set range. Otherwise, the product is not acceptable. Product specifications, or tolerance limits, are usually established by design engineers or product design specialists. For example, the specifications for the width of a machine part may be specified as 15 inches ±.3. This means that the width of the part should be 15 inches, though it is acceptable if it falls within the limits of 14.7 inches and 15.3 inches.

Process capability: The ability of a production process to meet or exceed pre-set specifications. Process capability thus involves evaluating process variability relative to pre-set product specifications in order to determine whether the process is capable of producing an acceptable product.

Measuring Process Capability

Process capability is measured by the process capability index, Cp, which is computed as the ratio of the specification width to the width of the process variability:

$$Cp = \text{Specification width} / \text{process width}$$

$$= (USL - LSL) / 6 * \sigma$$

Where the specification width is the difference between the upper specification limit (USL) and the lower specification limit (LSL) of the process. The process width is computed as 6 standard deviations (6 σ) of the process being monitored. The reason we use 6 σ is that most of the process measurement (99.74 percent) falls within ±3 standard deviations, which is a total of 6 standard deviations.

There are three possible ranges of values for Cp that also help us interpret its value:

$C_p = 1$: A value of Cp equal to 1 means that the process variability just meets specifications. We would then say that the process is minimally capable.

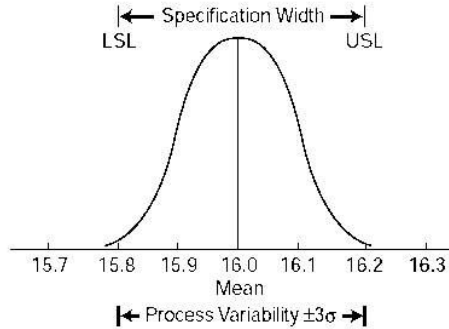


Figure: Process variability meets specification width

$C_p \leq 1$: A value of Cp below 1 means that the process variability is outside the range of specification. This means that the process is not capable of producing within specification and the process must be improved.

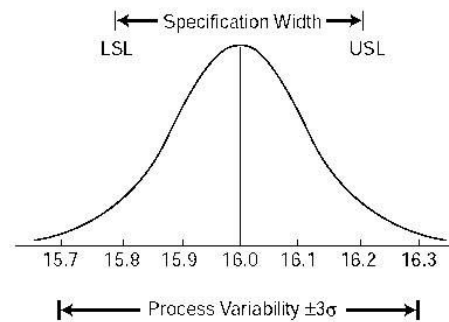


Figure: Process variability outside specification width

$C_p \geq 1$: A value of Cp above 1 means that the process variability is tighter than specifications and the process exceeds minimal capability.

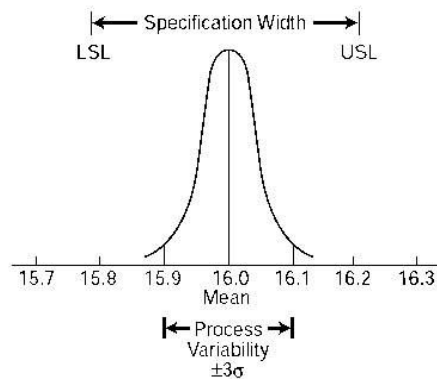


Figure: Process variability within specification width

A Cp value of 1 means that 99.74 percent of the products produced will fall within the specification limits. This also means that .26 percent (100% - 99.74%) of the products will not be acceptable. Although this percentage sounds very small, when we think of it in terms of parts per million (ppm) we can see that it can still result in a lot of defects. The number .26 percent corresponds to 2600 parts per million (ppm) defective (0.0026 * 1,000,000).

Example

Three bottling machines at Cocoa Fizz are being evaluated for their capability:

Bottling Machine	Standard Deviation
A	.05
B	.1
C	.2

If specifications are set between 15.8 and 16.2 ounces, determine which of the machines are capable of producing within specifications.

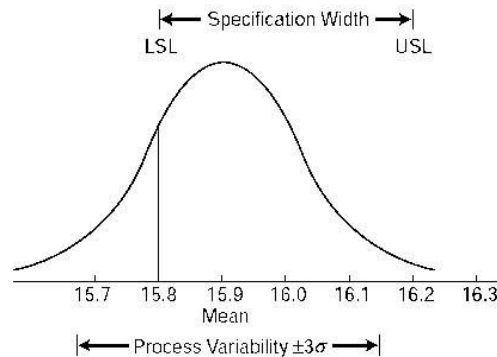
Solution

Bottling Machine	σ	USL-LSL	6σ	$C_p = \frac{USL - LSL}{6\sigma}$
A	.05	.4	.3	1.33
B	.1	.4	.6	.67
C	.2	.4	1.2	.33

Looking at the Cp values, only machine A is capable of filling bottles within specifications, because it is the only machine that has a Cp value at or above 1.

Cp is valuable in measuring process capability. However, it has one shortcoming: it assumes that process variability is centered on the specification range. Unfortunately, this is not always the case. Figure below shows data from the Cocoa Fizz example. In the figure the specification limits are set between 15.8 and 16.2 ounces, with a mean of 16.0 ounces. However, the process variation is not centered; it has a mean of 15.9 ounces. Because of this, a certain proportion of products will fall outside the specification range.

The problem illustrated in figure is not uncommon, but it can lead to mistakes in the computation of the Cp measure. Because of this, another measure for process capability is used more frequently:



Process variability not centered across specification width

$$C_{pk} = \min \left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right)$$

Where

μ = the mean of the process

σ = the standard deviation of the process

This measure of process capability helps us address a possible lack of centering of the process over the specification range. To use this measure, the process capability of each half of the normal distribution is computed and the minimum of the two is used.

Example

Compute the C_{pk} measure of process capability for the following machine and interpret the findings. What value would you have obtained with the C_p measure?

Machine Data:

$$USL = 110$$

$$LSL = 50$$

$$\text{Process } \sigma = 10$$

$$\text{Process } \mu = 60$$

Solution:

$$C_{pk} = \min \left(\frac{110-60}{3 \cdot 10}, \frac{60-50}{3 \cdot 10} \right) \\ = \min (1.67, \quad 0.33) = 0.33$$

This means that the process is not capable. The C_p measure of process capability gives us the following measure:

$$C_p = \frac{USL-LSL}{6 \cdot \sigma} \\ = \frac{110-50}{6 \cdot 10} \\ = 1$$

C_p leading us to believe that the process is capable. The reason for the difference in the measures is that the process is not centered on the specification range.