

Unit-5

Optics

Syllabus:

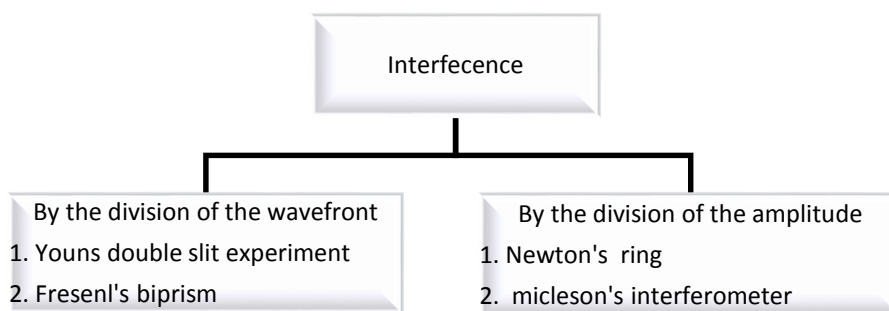
Interference, division of amplitude & division of wave front, double slit experiment, thin film interference, Newton Ring Experiment. Diffraction: Difference between interference and diffraction, types of diffraction, single slit, double slit & n-slit diffraction, Resolving power of grating.

Interface:

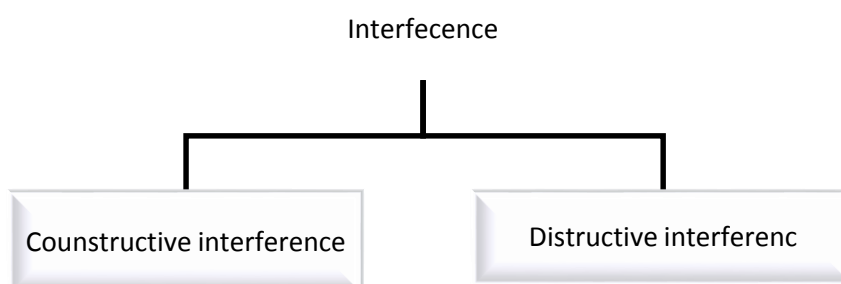
When two waves of approximately same amplitude and frequency going in the same direction in the same medium, generally coming from the same source, then the intensity of light at different places will be different. This phenomenon of light is known as interference.

Interference can be obtained by two ways:

Interference may be of two types:



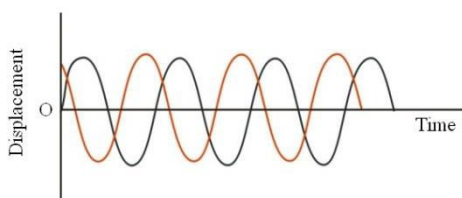
Figure(1): Interference Hierarchy tree



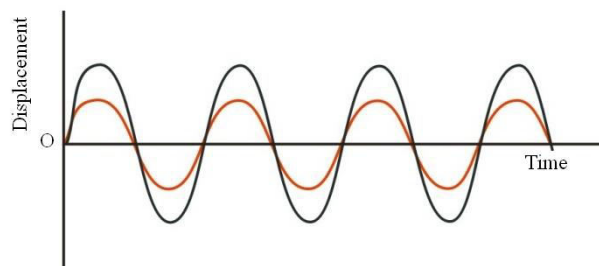
Figure(2): types of Interference

Constructive Interference:

Locus of all the points where the crest of one wave falls on the crest or the through of the one wave falls on the through of the other, the resultant amplitude is the sum of the individual waves. So the constructive interference takes place at those points and the intensity at these points will be maximum.



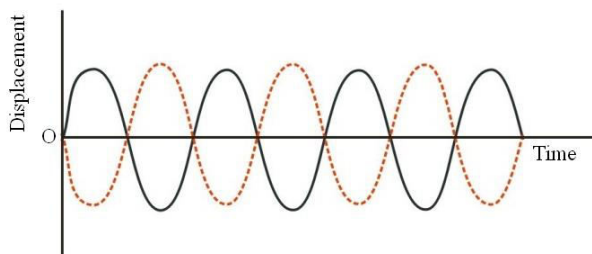
Figure(3): Constant Phase difference



Figure(4): Waves in same phase

Destructive Interference:

Locus of the points where the crest of one wave falls on the trough of the other wave the resultant intensity become the difference of the waves and at these places the intensity become minimum. At these points destructive interference will take place.

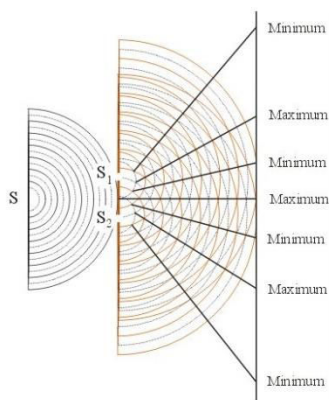


Figure(5): Waves opposite phase

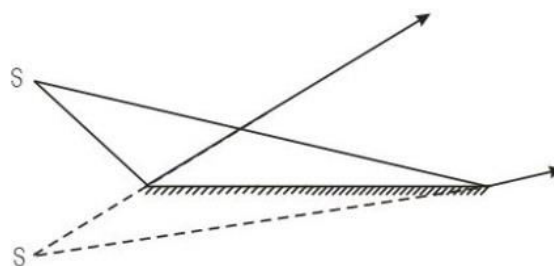
Coherent sources:

Two sources are said to be the coherent if they emit continuous light waves of the exactly same frequency/wavelength, nearly same amplitude and having sharply define phase difference that remains constant with the time.

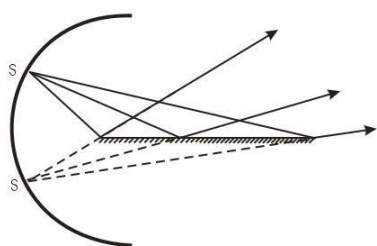
In practice it is impossible to have two independent coherent sources. For experimental purpose virtual sources formed by a single source and acts as coherent sources.



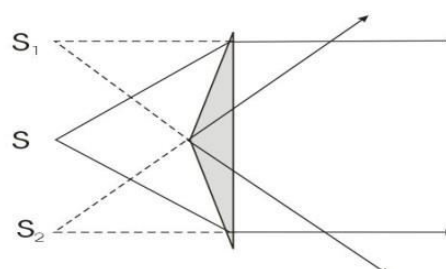
Figure(6): Young's Double Slit experiment



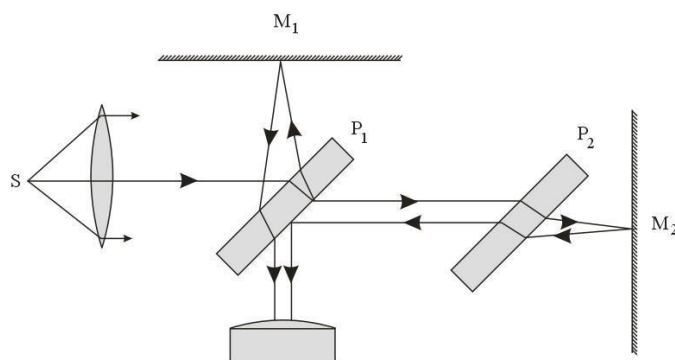
Figure(7): Lloyd's Mirror



Figure(8): Fresnel double mirror



Figure(9): Fresnel's bi-prism



Figure(10): Michelson's Interferometer

Relation between phase difference and path difference:

The difference between optical paths of two rays which are in constant phase difference with each other is known as the path difference.

Suppose for a path difference Δ the phase difference is ϕ

So

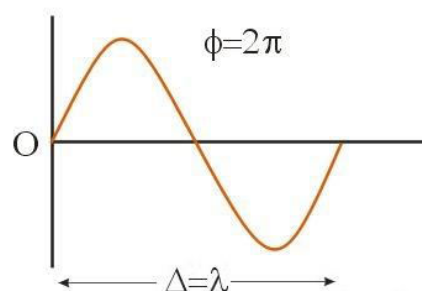
$$\phi = 2\pi \dots\dots\dots(1)$$

$$\Delta = \lambda \dots\dots\dots(2)$$

by equation (1) and (2)

$$\frac{\phi}{\Delta} = \frac{2\pi}{\lambda}$$

$$\phi = \frac{2\pi}{\lambda} \Delta$$



Figure(11): Phase and path difference

Principal of superposition:

When two or more waves reaches at the same point of a medium then the displacement at that point becomes the vector sum of displacement produced by the individual waves.

i.e.

$$Y = Y_1 + Y_2 + Y_3 + \dots$$

Mathematical treatment of interference:

Let two waves of amplitude a_1 and a_2 and angular frequency ω super impose and re-unit at a point after traveling different path S_1P and S_2P , let the phase difference of these two waves is ϕ

If y_1 and y_2 are two waves then

$$y_1 = a_1 \sin(\omega t) \quad \dots\dots\dots(1)$$

$$y_2 = a_2 \sin(\omega t + \phi) \quad \dots\dots\dots(2)$$

By the principle of superposition of waves, the resultant waves will be

$$y = y_1 + y_2$$

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$\because \sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$y = a_1 \sin \omega t + a_2 [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$y = a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$y = \sin \omega t [a_1 + a_2 \cos \phi] + a_2 \sin \phi \cos \omega t \quad \dots\dots\dots(3)$$

$$\text{Let } A \cos \theta = a_1 + a_2 \cos \phi \quad \dots\dots\dots(4)$$

$$A \sin \theta = a_2 \sin \phi \quad \dots\dots\dots(5)$$

by the equation (3), (4) and (5) we get-

$$y = \sin \omega t \cdot A \cos \theta + \cos \omega t \cdot A \sin \theta$$

$$y = A[\sin \omega t \cos \theta + \cos \omega t \sin \theta]$$

$$y = A \sin(\omega t + \theta) \quad \dots\dots\dots(6)$$

Here A and θ are constant and can be given by equation (4) and (5) as

$$(a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2 = A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

$$a_1^2 + 2a_1 a_2 \cos \phi + a_2^2 \cos^2 \phi + a_2^2 \sin^2 \phi = A^2 [\cos^2 \theta + \sin^2 \theta]$$

$$a_1^2 + 2a_1 a_2 \cos \phi + a_2^2 (\cos^2 \phi + \sin^2 \phi) = A^2 [\cos^2 \theta + \sin^2 \theta]$$

$$(a_1^2 + a_2^2) + 2a_1 a_2 \cos \phi = A^2$$

$$\because \cos^2 \phi + \sin^2 \phi = 1$$

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

Now the resultant intensity at any point is given as $I \propto A^2$ for simplicity let

$$I = A^2$$

So

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

Condition for maxima:

For maximum intensity

$$\cos \phi = 1$$

then $\phi = 2n\pi$

This is the condition for constructive interference in terms of phase(ϕ)

Then by equation by (6)

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi$$

$$I = (a_1 + a_2)^2$$

So the path difference

$$\Delta = \frac{\lambda}{2\pi} \times \phi$$

$$\Delta = \frac{\lambda}{2\pi} \times 2n\pi$$

$$\Delta = n\lambda$$

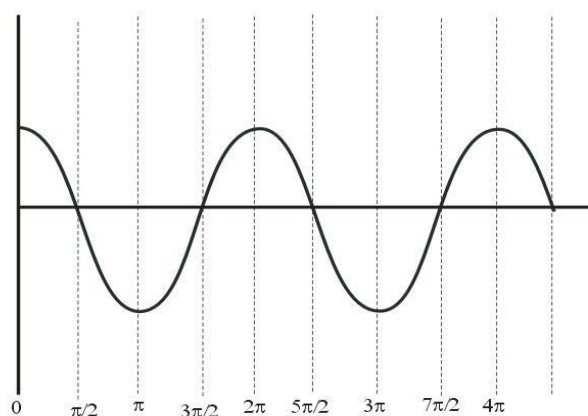
$$\Delta = 2n \frac{\lambda}{2}$$

I.e. the path difference is the even

multiple of $\frac{\lambda}{2}$, this is the condition of

constructive interference in terms of

path difference (Δ)



Figure(12): cosine curve

Condition for the minima:

Again the intensity will be minimum when-

$$\cos \phi = -1$$

$$\text{then } \phi = (2n + 1)\pi$$

This is the condition for destructive interference in terms of phase(ϕ)

Then by equation (6)

$$I = a_1^2 + a_2^2 - 2a_1a_2 \cos \phi$$

$$I = (a_1 - a_2)^2$$

And path difference

$$\Delta = \frac{\lambda}{2\pi} \times \phi$$

$$\Delta = \frac{\lambda}{2\pi} (2n + 1)\pi$$

$$\Delta = (2n + 1) \frac{\lambda}{2}$$

i.e. the odd multiple of the half wavelength, this is the condition of destructive interference in terms of path difference (Δ)

Now the average Intensity:

$$\begin{aligned}
 I_{av} &= \frac{\int_0^{2\pi} I d\phi}{\int_0^{2\pi} d\phi} \\
 I_{av} &= \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2 \cos \phi) d\phi}{\int_0^{2\pi} d\phi} \\
 I_{av} &= \frac{[a_1^2\phi + a_2^2\phi + 2a_1a_2 \sin \phi]_0^{2\pi}}{[\phi]_0^{2\pi}} \\
 I_{av} &= \frac{(a_1^2 + a_2^2) \cdot 2\pi}{2\pi} \\
 I_{av} &= a_1^2 + a_2^2 \\
 I_{av} &= I_1^2 + I_2^2
 \end{aligned}$$

The average intensity is the average of the maximum and minimum intensities. It can be given by-

Now if $a_1 = a_2 = a$ then,

$$I_{av} = 2a^2$$

The average intensity is equal to the sum of the separate intensities. Whatever the intensity disappears at the minima is actually appears at the maxima. Thus there is no violation of the law of conservation of energy in the phenomena of interference.

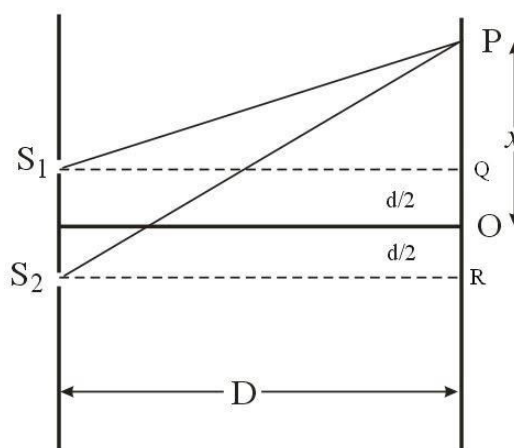
Condition for the sustained interference of light.

1. Two sources of light must be coherent.
2. Difference in the amplitudes of the two waves must be small.
3. Sources should be narrow or point source.
4. The separation between two sources should as small as possible.
5. If the interfering waves are polarised then the plane of polarisation must be same.
6. The sources should be monochromatic.
7. Interfering waves from two coherent sources of light should travel in the same direction.

Fringe width:

Consider a narrow monochromatic source S and two parallel narrow slits S_1 and S_2 very close together and equidistant from S . Let d be the distance between two slits S_1 and S_2 and D be the distance of screen from coherent source. The path difference between the rays reaching from S_1 and S_2 to O is zero so the point O has maximum intensity.

Considering a point P at a distance x from O . The wave reaches at the point P from S_1 and S_2 hence $PQ = \left(x - \frac{d}{2}\right)$ and $PR = \left(\frac{d}{2}\right) +$



Figure(13): Measurement of fringe width

$$\begin{aligned}
 S_2P^2 - S_1P^2 &= \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right] - \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right] \\
 (S_2P + S_1P)(S_2P - S_1P) &= \left[D^2 + x^2 + \frac{d^2}{4} - 2x\frac{d}{2} \right] - \left[D^2 + x^2 + \frac{d^2}{4} + 2x\frac{d}{2} \right] \\
 (S_2P + S_1P)(S_2P - S_1P) &= D^2 + x^2 + \frac{d^2}{4} - 2x\frac{d}{2} - D^2 - x^2 - \frac{d^2}{4} - 2x\frac{d}{2} \\
 (S_2P + S_1P)(S_2P - S_1P) &= -2dx \\
 S_2P - S_1P &= \frac{2dx}{(S_2P + S_1P)}
 \end{aligned}$$

Now from the figure

If the point P is very close to point O

$$\text{so } S_2P - S_1P \approx S_2P \approx S_1P \approx D$$

$$\begin{aligned}
 \Delta &= \frac{2dx}{(D + D)} \\
 \Delta &= \frac{2dx}{2D}
 \end{aligned}$$

$$\Delta = \frac{dx}{D}$$

1. **Bright Fringes:** For bright fringes the path difference is the integer multiple of the $\frac{\lambda}{2}$ i.e.

$$\frac{xd}{D} = 2n \cdot \frac{\lambda}{2}$$

$$\frac{xd}{D} = n\lambda$$

$$x = \frac{n\lambda D}{d}$$

This equation gives the distance of the bright fringes from the point O . Hence for the n^{th} bright fringe (replacing x by x_n)

$$x_n = \frac{n\lambda D}{d}$$

For next bright fringe

$$x_{n+1} = \frac{(n+1)\lambda D}{d}$$

Therefore the distance between any two consecutive bright fringes

$$x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d}$$

2. **Dark Fringes:** For dark fringes the path difference is an odd multiple of $\frac{\lambda}{2}$

So

$$\frac{dx}{D} = (2n+1) \frac{\lambda}{2}$$

$$x = \frac{(2n+1)\lambda D}{2d}$$

Hence the n^{th} dark fringe (replacing x by x_n)

$$x_n = \frac{(2n+1)\lambda D}{2d}$$

And for the $(n+1)^{th}$ dark fringe

$$x_{n+1} = \frac{[2(n+1)+1]\lambda D}{2d}$$

$$x_{n+1} = \frac{(2n+3)\lambda D}{2d}$$

Therefore the distance between two consecutive dark fringes

$$x_{n+1} - x_n = \frac{(2n+3)\lambda D}{2d} - \frac{(2n+1)\lambda D}{2d}$$

$$x_{n+1} - x_n = \frac{\lambda D}{2d} [2n + 3 - 2n - 1]$$

$$x_{n+1} - x_n = \frac{\lambda D}{2d} \cdot 2$$

$$\beta' = \frac{\lambda D}{d}$$

As the distance between two consecutive bright or dark fringes is same and is called fringe width and denoted by β .

$$\beta = \frac{D\lambda}{d}$$

- The fringe width is directly proportional to the wavelength of the light used i.e. $\beta \propto \lambda$
- The fringe width is directly proportional to the distance of the slits from the screen i.e. $\beta \propto D$
- The fringe width is inversely proportional to the distance between the slits i.e. $\beta \propto \frac{1}{d}$

Shape of the interference fringes:

Actually these interfering fringes are hyperbolic in shape, but the eccentricity of fringes is quite large and hence these hyperbolic fringes appear more or less straight lines.

Angular Fringe Width:

The angular fringe width is defined as the angular separation between consecutive or dark fringes and is denoted by θ .

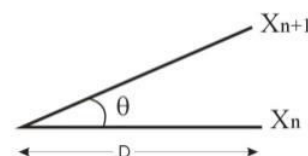


Figure (14): Angular fringe width

$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\theta = \theta_{n+1} - \theta_n$$

$$\theta = \frac{x_{n+1}}{D} - \frac{x_n}{D}$$

$$\theta = \frac{(x_{n+1} - x_n)}{D}$$

$$\theta = \frac{\beta}{D}$$

But $\beta = \frac{\lambda D}{d}$

so

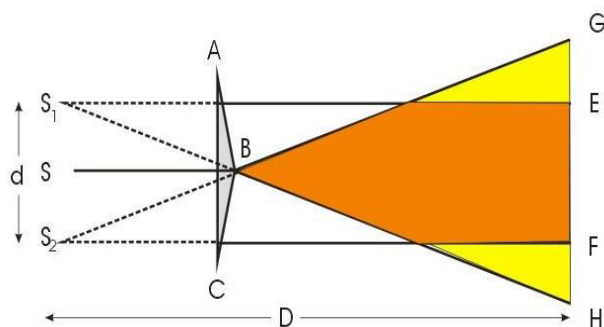
$$\theta = \frac{\lambda}{d}$$

Fresnel's Biprism:

The prism is a device to obtain two coherent sources to produce sustained interference.

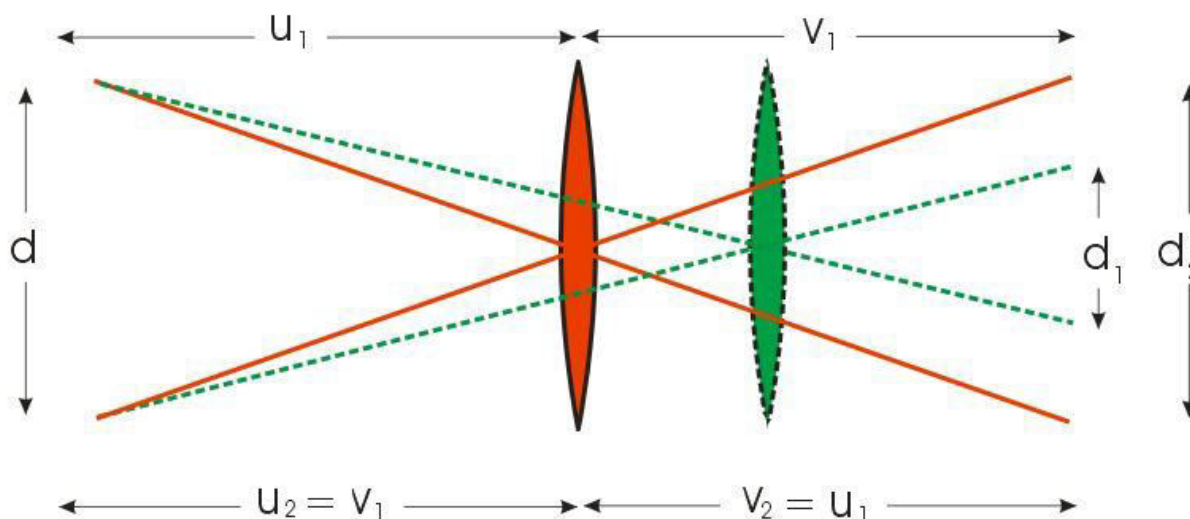
Fresnel used a biprism to show the phenomenon of interference. A biprism is usually a combination of two prisms placed base to base. In fact this combination is obtained from an optically plane glass plate by proper grinding and polishing. The obtuse angle of the prism is about 179° and other angles are about $30'$ each.

To show the phenomenon of interference a horizontal section of the apparatus is shown in the figure.



Figure(15): Fresnel's Biprism

Measurement of d



Figure(16): Measurement of d by displacement of lens

A bi-convex lens of short focal length is mounted between the bi-prism P and the eyepiece by moving the lens along length of bench, two positions L_1 and L_2 are obtained such as for which the image of sources formed at the same place.

For L_1 position
$$\frac{d_1}{d} = \frac{v_1}{u_1} \quad \dots\dots\dots(1)$$

For L_2 position
$$\frac{d_2}{d} = \frac{v_2}{u_2}$$

$$\frac{d_2}{d} = \frac{u_1}{v_1} \quad [\because v_2 = u_1 \text{ and } u_2 = v_1] \quad \dots\dots\dots(2)$$

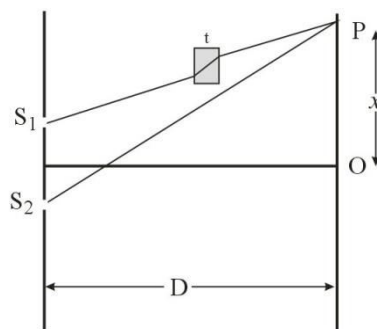
So on multiplying (1) & (2) we get

$$\frac{d_1 d_2}{(d)^2} = \frac{v_1}{u_1} \times \frac{u_1}{v_1}$$

$$(d)^2 = d_1 d_2$$

$$d = \sqrt{d_1 d_2}$$

Determination of the thickness of a thin sheet of transparent material:



Figure(17): Shift in fringes on introducing the thin film

Distance travelled by the light in air = S_1P (by the velocity c)

Distance travelled by the light in film = S_1Q (by the velocity c_g)

Time taken by the light to cover this distance

$$T = \frac{S_1P}{c} + \frac{t}{c_g}$$

But $\mu = \frac{c}{c_g}$ then $c_g = \frac{c}{\mu}$

So we have

$$T = \frac{S_1P}{c} + \frac{t}{\left(\frac{c}{\mu}\right)}$$

$$T = \frac{S_1P}{c} + \frac{\mu t}{c}$$

$$T = \frac{S_1P}{c} + \mu t$$

$$T = \frac{S_1P + t(\mu - 1)}{c}$$

Thus the path S_1P is equivalent to an air path $S_1P + (\mu - 1)t$

Now the path difference at

$$\Delta = \text{path } S_2P \text{ (in air)} - \text{path } S_1P \text{ (in air)}$$

$$\Delta = S_2P - [S_1P + (\mu - 1)t]$$

$$\Delta = (S_2P - S_1P) - (\mu - 1)t$$

But $S_2P - S_1P = \frac{xd}{D}$

So we have

$$\Delta = \frac{xd}{D} - (\mu - 1)t \quad \dots \dots (1)$$

but for n^{th} maxima,

$$\Delta = n\lambda \quad \dots \dots (2)$$

So by equation (1) and (2)

$$\begin{aligned} \frac{x_n d}{D} - (\mu - 1)t &= n\lambda \\ x_n &= \frac{D}{d}[n\lambda - (\mu - 1)t] \quad \dots \dots (3) \end{aligned}$$

Where x_n is the position of the n^{th} maxima

Now in absence of the plate i.e. $t = 0$

$$\text{The } n^{\text{th}} \text{ maxima} \quad x'_n = \left[\frac{Dn\lambda}{d} \right] \quad (\text{in the absence of } t) \quad \dots \dots (4)$$

1. Displacement of the fringes:

If S denotes the displacement of the n^{th} maxima by introducing the mica sheet, then

$$\begin{aligned} x_n - x'_n &= \frac{D}{d}[n\lambda - (\mu - 1)t] - \frac{Dn\lambda}{d} \\ S &= \frac{Dn\lambda}{d} + \frac{D}{d}(\mu - 1)t - \frac{Dn\lambda}{d} \\ S &= \frac{D}{d}(\mu - 1)t \end{aligned}$$

This equation is free from n so the displacement of each maxima will be same.

2. Thickness of mica sheet:

The displacement S of any maxima by introducing a mica sheet of thickness t is given by

$$t = \frac{S \times d}{D(\mu - 1)}$$

3. Refractive index of the material of prism:

Once if we know the displacement of the fringes and thickness of the film we can calculate the refractive index of the material of the film as-

$$\begin{aligned} (\mu - 1) &= \frac{S \times d}{Dt} \\ \mu &= \frac{S \times d}{Dt} + 1 \end{aligned}$$

Stoke's treatment of phase change:

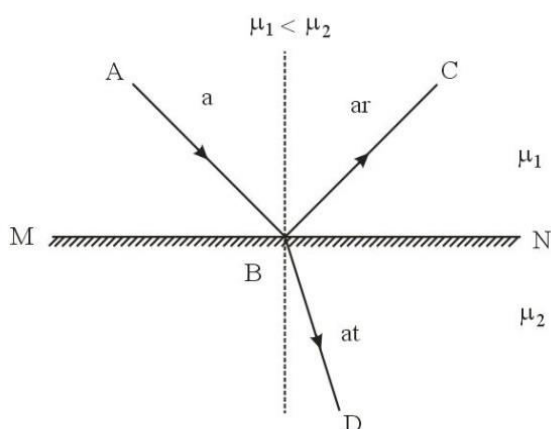
When a light wave is reflect from the surface of an optically dens medium, it suffers a phase change of π i. e. a path difference of $\frac{\lambda}{2}$

Let MN is an interface separating the denser medium (below i.e. glass) to rare medium (above i.e. air) it. A ray of light AB of amplitude " a " incident on the interface MN is partially reflecte along the path BC and patially refracted into the denser medium along BD . Let r is the coefficient of reflection and t is the coefficient of transmission then the amplitude of reflected and transmitted wave will be ' ar ' and at respectively.

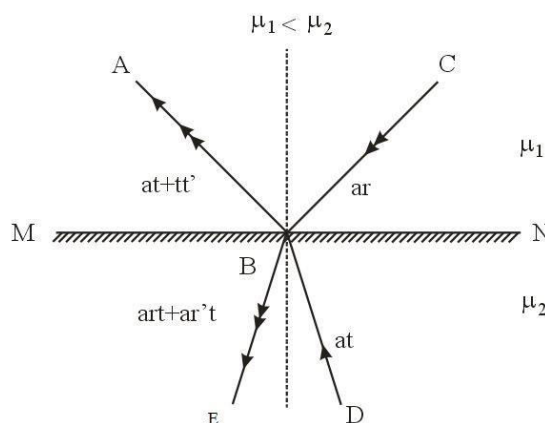
Then in case of no absorption of light

$$r + t = 1$$

Now if the reflected and refracted rays are reversed the resultant should have the same amplitude ' a ' as that of the incident ray



Figure(18): Reflection and refraction through a surface



Figure(19): Ray diagram on reversing the direction of incidence

When CB is reversed it is partly reflected along BA and partially refracted along BE as shown in figure.

Similarly when the ray DB is revesed it is partly refracted along BA and partially reflected along BE . Now the content along BE should be zero and that along BA should be equal to a i. e.

$$art + \cancel{atr} = 0 \quad \dots \dots \dots (1) \quad \dots \dots \dots$$

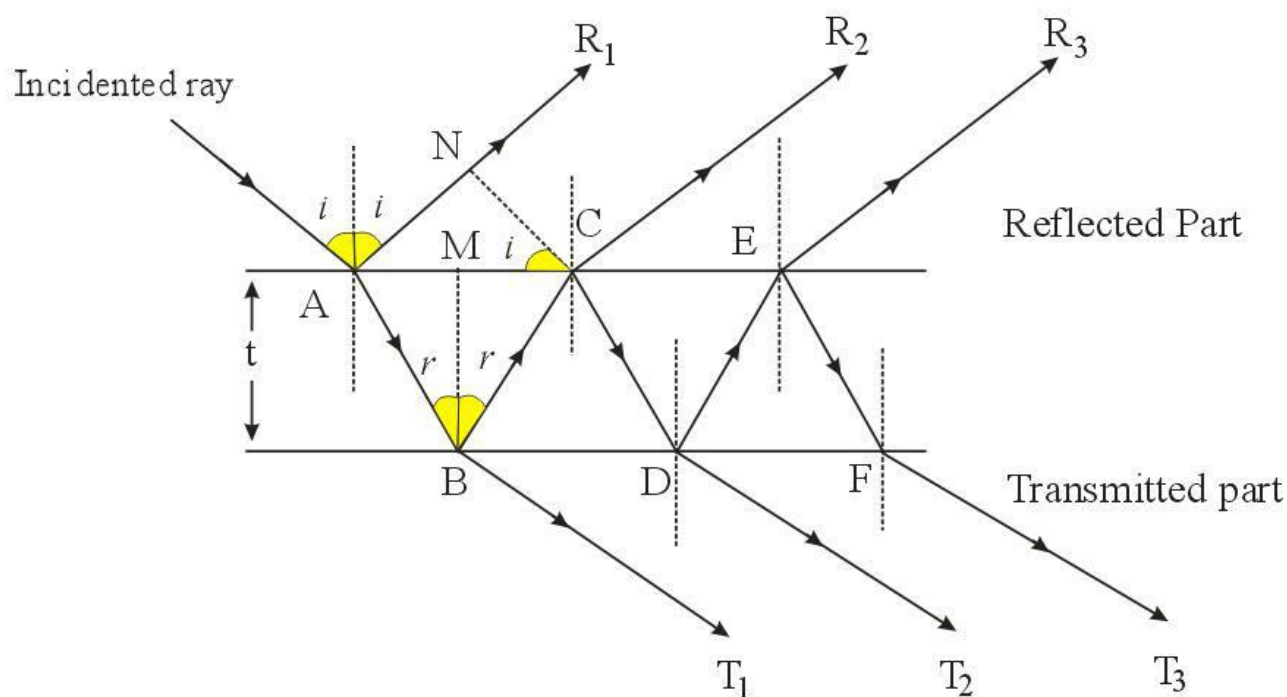
$$art = -atr'$$

$$r = -r'$$

This equation indicates displacement in the opposite direction so according to Stoke's law, when a light wave coming from a rare medium an additional phase π is introduced in it.

Interference in thin film:

Consider a thin film of equal thickness t and refractive index $\mu (> 1)$. A monochromatic light ray SA incident at angle i is partially reflected and partially transmitted as shown in figure



Figure(20): Reflection and transmission of light through a thin film

Reflected system:

In reflected system two waves AR_1 & CR_2 are in the position to interfere so the path difference between AR_1 & BR_2

$$\Delta = (\text{path } ABC)_{\text{in film}} - (\text{path } AN)_{\text{in air}}$$

$$\Delta = \mu(AB + BC) - AN$$

Now from figure it is clear that $AB = BC$

$$\Delta = \mu(2BC) - AN$$

But from ΔBMC

$$\frac{BC}{BM} = \sec r$$

Then

$$BC = BM \sec r$$

$$BC = t \sec r$$

And from ΔACN

$$\frac{AN}{AC} = \sin i$$

$$AN = AC \sin i$$

But $AC = AM + MC$

$$AN = (AM + MC) \sin i \quad \dots\dots\dots (1)$$

But from ΔABM and ΔBMC

$$\frac{AM}{BM} = \tan r \Rightarrow AM = BM \tan r \Rightarrow AM$$

and $\frac{MC}{BM} = \tan r \Rightarrow MC = BM \tan r \Rightarrow MC =$

but $AM = t \tan r$ and $MC = t$ \dots\dots\dots (2)

putting the value of AM and MC in (1) from (2) so we have

$$AN = (t \tan r + t) \sin i$$

$$AN = 2t \tan r \sin i$$

$$AN = 2t \frac{\sin r}{\cos r} \sin i$$

Multiplying and dividing by $\sin r$ we get

$$AN = 2t \frac{\sin r}{\cos r} \frac{\sin i}{\sin r} \sin r$$

$$AN = 2t \frac{\sin^2 r}{\cos r} \mu$$

$$AN = 2\mu t \frac{\sin^2 r}{\cos r}$$

Therefore $\Delta = \mu(2BC) - AN$

Putting the value of BC and AN , we get

$$\Delta = 2\mu t \sec r - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\Delta = 2\mu t \left(\sec r - \frac{\sin^2 r}{\cos r} \right)$$

$$\Delta = 2\mu t \left(\frac{1}{\cos r} - \frac{\sin^2 r}{\cos r} \right)$$

$$\Delta = 2\mu t \left(\frac{1 - \sin^2 r}{\cos r} \right)$$

$$\Delta = 2\mu t \left(\frac{\cos^2 r}{\cos r} \right)$$

$$\Delta = 2\mu t \cos r \quad \dots\dots\dots (3)$$

$\therefore \frac{\mu \sin i}{\sin r}$
--

A. In Reflected system:

The ray AR_1 undergoes a reflection from the denser medium so an additional path difference of $\frac{\lambda}{2}$ must be added, then

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$$

1. Condition for constructive interference:

For constructive interference $\Delta = n\lambda$

So

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda - \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n - 1)\frac{\lambda}{2} \quad (\text{where } n = 1, 2, 3, \dots)$$

2. Condition for the destructive interference:

For destructive interference $\Delta = (2n + 1)\frac{\lambda}{2}$

So

$$2\mu t \cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \quad (\text{where } n = 1, 2, 3, \dots)$$

B. In transmitted system:

In the transmitted system there will be no additional path difference so

$$\Delta = 2\mu t \cos r$$

1. Condition for constructive interference: Condition for constructive interference is $\Delta = n\lambda$

then $2\mu t \cos r = n\lambda$ (Where $n = 0, 1, 2, 3, \dots$)

2. Condition for destructive interference: Condition for the destructive interference is

$$\Delta = (2n + 1)\frac{\lambda}{2} \quad (\text{Where } n = 1, 2, 3, \dots)$$

then $2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$

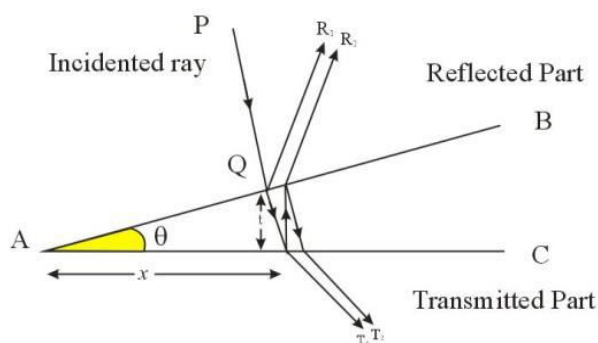
So the reflected and transmitted interference patterns are complementary.

Colour in thick film:

A thick film does not show any colour in the reflected system when illuminated with an extended source of light.

Wedge shape film:

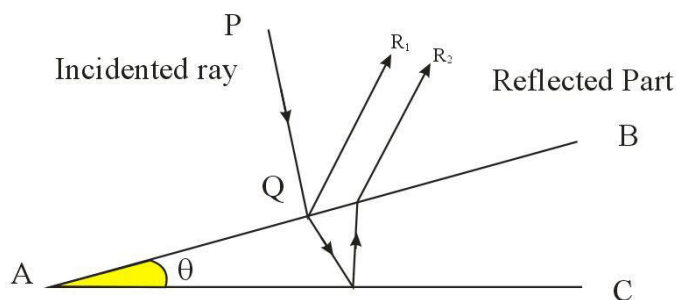
A wedge shape film is one whose surfaces are inclined at a certain small angle. Figure shows a thin wedge shape film of refractive index μ bounded by two plane surfaces AB and CD inclined at an angle θ . Let a parallel beam of monochromatic light falls on the upper surface AB normally and the surface is viewed in the reflected and refracted system then alternate dark and bright fringes becomes visible.



Figure(21): Reflection and refraction through a wedge shape film

Let the light is incident nearly normally at a point Q on the film, the path difference between the rays reflected at the upper and lower surface is $2\mu t$ ($\because r \approx 0$) where t is the thickness of the film at Q .

Reflected system:



Figure(22): Reflection through a wedge shape film

The condition for the maximum intensity (bright fringes):

In the reflected system according to the Stokes treatment an additional path difference of $\frac{\lambda}{2}$ is introduced in the ray reflected from the upper surface. Hence the effective path difference between the two rays will be $\Delta = 2\mu t - \frac{\lambda}{2}$ and the condition for the bright fringes is $\Delta = n\lambda$

$$\text{So} \quad 2\mu t - \frac{\lambda}{2} = n\lambda$$

$$2\mu t = n\lambda \frac{\lambda}{2}$$

$$2\mu t = (2n - 1) \frac{\lambda}{2}$$

The condition for the minimum intensity (Dark fringes):

The condition for the destructive interference is $\Delta = (2n - 1) \frac{\lambda}{2}$

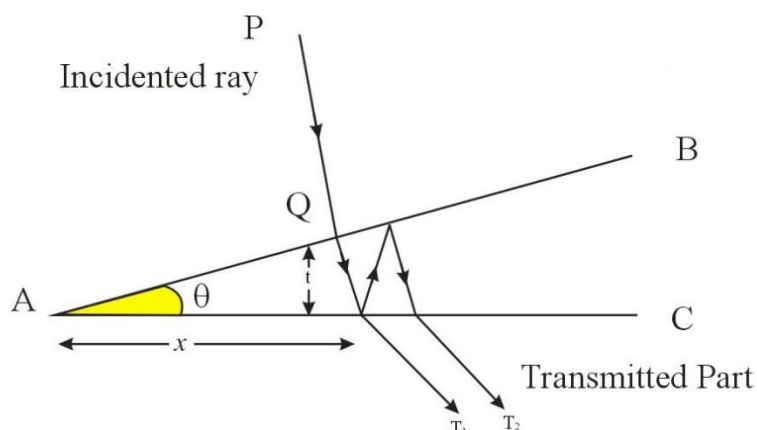
$$2\mu t \frac{\lambda}{2} = (2n - 1) \frac{\lambda}{2}$$

$$2\mu t \frac{\lambda}{2} = n\lambda \frac{\lambda}{2}$$

$$2\mu t = n\lambda \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$2\mu t = n\lambda$$

Transmitted System:



Figure(23): Refraction through a wedge shape film

In the transmitted system there will be no additional path difference so the effective path difference will be $\Delta = 2\mu t$

The condition for the maximum intensity (bright fringes):

The condition for the maxima is given as $\Delta = n\lambda$

so
$$2\mu t = n\lambda$$

The condition for the minimum intensity (Dark fringes):

The condition for the minima in interference is $\Delta = (2n + 1) \frac{\lambda}{2}$

$$\text{So} \quad 2\mu t = (2n + 1) \frac{\lambda}{2}$$

Fringe width:

For n^{th} dark fringe let this fringe observed at a distance x_n from the edge, where the thickness of fringe is

t_n From figure (23-B) it is clear that $t_n = x_n \theta$

$$\text{then} \quad t_n = x_n \theta$$

$$\text{So} \quad 2\mu t_n = n\lambda$$

$$\text{So} \quad 2\mu x_n \theta = n\lambda \quad \dots\dots(1)$$

Similarly for $(n + 1)^{\text{th}}$ fringe

$$2\mu x_{n+1} \theta = (n + 1)\lambda \quad \dots\dots(2)$$

By equation (1) and (2)

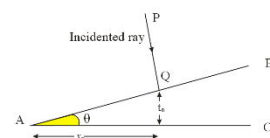
$$2\mu x_{n+1} \theta - 2\mu x_n \theta = (n + 1)\lambda - n\lambda$$

$$2\mu(x_{n+1} - x_n) \theta = \lambda$$

$$\text{But} \quad x_{n+1} - x_n = \beta$$

$$\text{so} \quad 2\mu \beta \theta = \lambda$$

$$\beta = \frac{\lambda}{2\mu \theta}$$

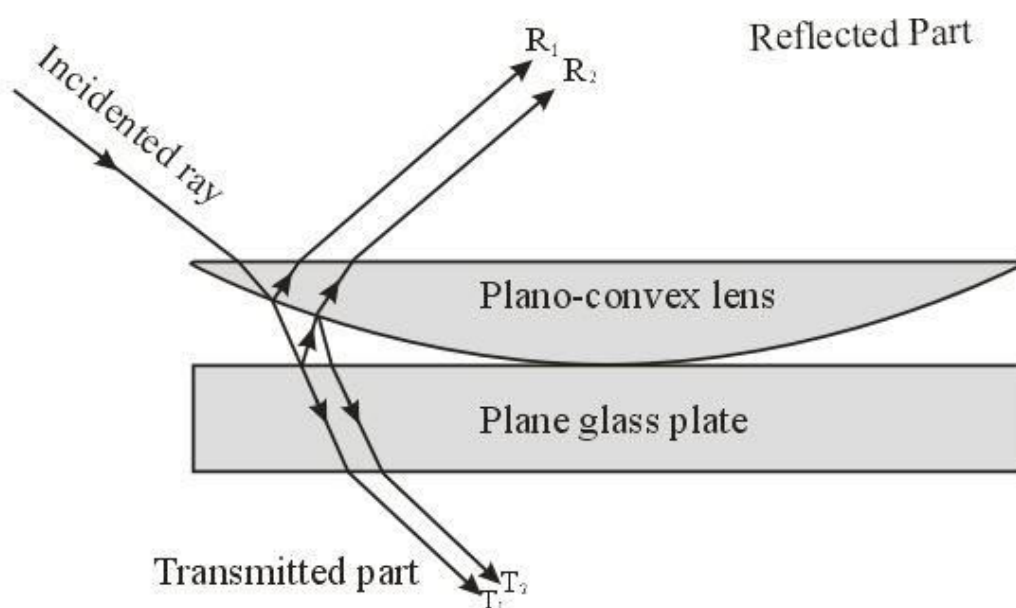


Figure(23-B)

Newton's Ring:

Formation of Newton's Ring:

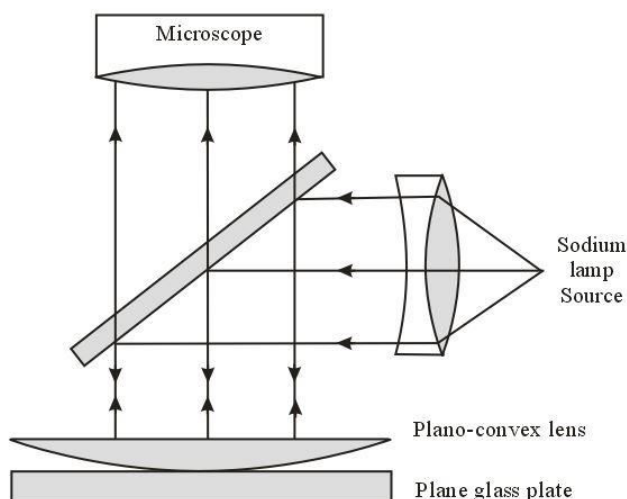
When a Plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, an air film of gradually increasing thickness is formed between the upper surface of the plane glass plate and the lower surface of the Plano-convex lens. If a monochromatic beam of light is allowed to fall normally on the upper surface of the film then, alternative bright and dark concentric fringes with their centre dark are formed. These fringes or rings are known as the Newton's rings.



Figure(24): Formation of the Newton's rings

Experimental arrangement:

The experimental arrangement is shown in the figure. Light rays reflected upwards from the air film, superimpose each other and interference takes place, due to which the alternative bright and dark concentric rings are formed those can be seen by the telescope.



Figure(25): Newton's ring experimental arrangement

The fringes are circular because the air film is symmetrical about the point of contact of the lens with the plane glass plate.

Theory:

The rings are formed both in reflected and refracted part.

Reflected Part:

As the films are obtained in the reflected part the effective path difference between the interfering rays is given by

$$\Delta = 2\mu t \cos \frac{r}{2} + \frac{\lambda}{2} \quad \text{..... (1)}$$

Where μ is the refractive index of the film, t is the thickness of the film, r is the angle of incidence. The factor $\frac{\lambda}{2}$ is account for the phase change of π on reflection from the lower surface of the film. For air $\mu = 1$ and for normal incidence $r = 0$ then

$$\Delta = 2t \frac{\lambda}{2} \quad \text{..... (2)}$$

Central fringe:

At the centre i.e. at the point of contact $t = 0$

So
$$\Delta = \frac{\lambda}{2}$$

This is the condition for the minimum intensity, hence the central fringe will be dark.

For Constructive interference (i.e. maxima):

The condition for the constructive interference by thin film is given as

$$\Delta = n\lambda$$

$$\therefore 2t \frac{\lambda}{2} = n\lambda$$

$$2t = n\lambda \frac{\lambda}{2}$$

$$\text{Then } 2t = (2n - \frac{\lambda}{2})$$

It is the condition for constructive interference

Where $n = 1, 2, 3$

For destructive interference (i.e. minima):

The condition for the destructive interference by the thin film is given as $\Delta \neq (2n + \frac{\lambda}{2})$

$$\therefore 2t \frac{\lambda}{2} = (2n + \frac{\lambda}{2})$$

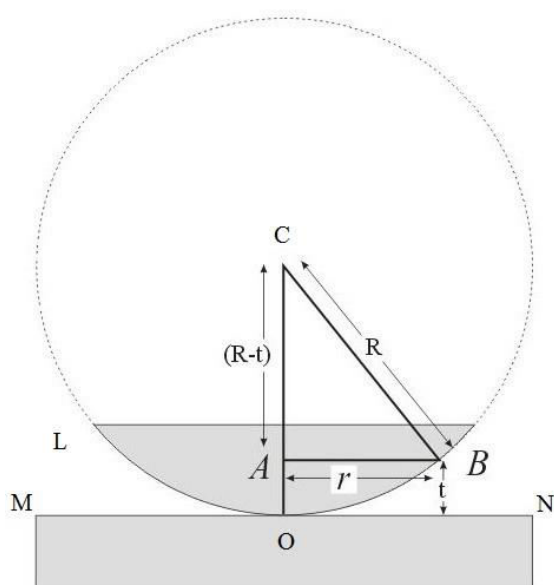
$$2t = n\lambda$$

It is the condition for minima

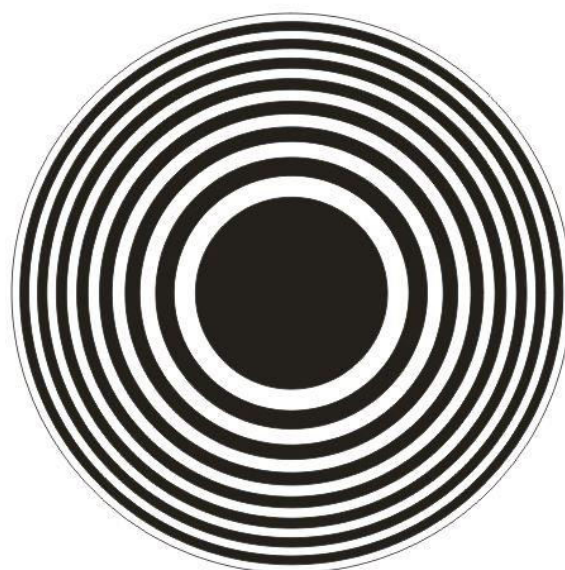
Where $n = 1, 2, 3$

Shape of the fringes:

As in air film t remains constant along the circle with its center at the point of contact, the fringes are in the form of the circles, since each film is the locus of the constant thickness of the air film. These fringes are known as the fringes of equal thickness.



Figure(26): Diameter of Newton's ring



Figure(27): Shape of the Newton's rings

So the diameter of the bright ring is proportional to the square root of the odd number.

Diameter of Bright ring:

Let L is the lens placed in the glass plate MN the point of contact is shown by O . Let R is the radius of the

curvature of the curved surface of the lens. Let r be the radius of the Newton's ring where the film thickness is t

from the right angle ΔCAB

$$R^2 = (R - t)^2 + r^2$$

$$R^2 = R^2 + t^2 - 2Rt + r^2$$

As the air film is very thin so t^2 can be neglected

$$0 = -2Rt + r^2$$

$$r^2 = 2Rt$$

$$2t = \frac{r^2}{R}$$

Substituting the value of $2t$ in the equation for bright ring i.e. $2t = (2n + 1) \frac{\lambda}{2}$

So

$$\frac{r^2}{R} = (2n + 1) \frac{\lambda}{2}$$

\therefore Radius of n^{th} bright ring

$$r_n = \sqrt{\frac{(2n + 1)\lambda R}{2}}$$

So the diameter of n^{th} bright ring

$$D_n = 2r_n$$

So

$$D_n = 2 \sqrt{\frac{(2n + 1)\lambda R}{2}}$$

$$D_n = \sqrt{(2n + 1)2\lambda R}$$

$$D_n = \sqrt{2\lambda R} \sqrt{2n + 1}$$

$$D_n \propto \sqrt{2n + 1}$$

Diameter of the Dark ring:

Condition for dark ring is

$$2t = n\lambda$$

And

$$2t = \frac{r^2}{R}$$

So on comparing these two equation we get

$$\frac{r^2}{R} = n\lambda$$

$$r^2 = nR\lambda$$

$$r = \sqrt{nR\lambda}$$

\therefore the diameter of the dark ring

$$D_n = 2r_n$$

$$D_n = 2\sqrt{nR\lambda}$$

$$D_n = \sqrt{4nR\lambda}$$

$$D_n = \sqrt{2n}\sqrt{2R\lambda}$$

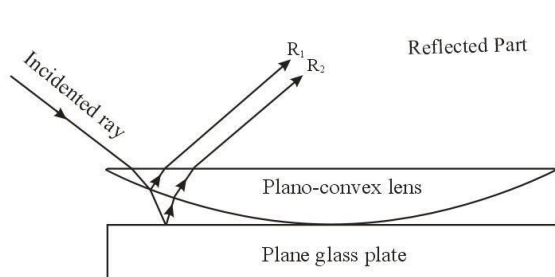
$$D_n \propto \sqrt{2n}$$

The diameter of dark ring is proportional to the square root of even number.

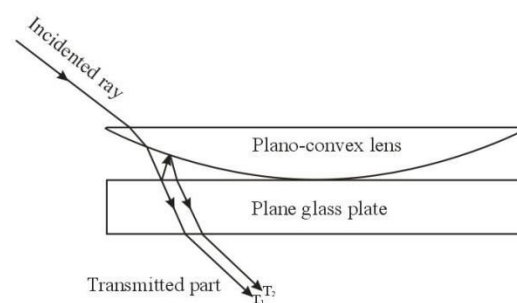
So, as we go far from the centre the thickness of the ring reduces, this limits the number of rings in any pattern that means infinite number of ring can-not be seen.

Newton's Rings in transmitted part:

In case of transmitted light, the effective path difference is $2\mu t \cos r$



Figure(25): Newton's ring reflection system



Figure(26): Newton's ring in Refracted system

Transmitted part:

Constructive interference:

$$2t = n\lambda$$

Destructives interference:

$$2t = (2n + 1)\frac{\lambda}{2}$$

S.N.	Name	Figure	Condition for Reflected part	Condition for transmitted part
1	Parallel Thin film		Constructive: (Maxima) $2\mu t \cos r = (2n - 1) \frac{\lambda}{2}$	$2\mu t \cos r = n\lambda$
2	Wedge shape film		Constructive: (Maxima) $2\mu t = (2n - 1) \frac{\lambda}{2}$	$2\mu t = n\lambda$
3	Newton's ring		Constructive: (Maxima) $2t = (2n - 1) \frac{\lambda}{2}$	$2t = n\lambda$
			Destructive: (Minima) $2t = n\lambda$	$2t = (2n - 1) \frac{\lambda}{2}$
			Destructive: (Minima) $2t = n\lambda$	$2t = (2n - 1) \frac{\lambda}{2}$

Applications of the Newton's Ring:

1. Determination of wavelength of light

Let D_n and D_{n+p} respectively the diameters of the n^{th} and $(n + p)^{th}$ dark rings where p is an integer. Then by equation

$$D_n^2 = 4nR\lambda \quad \dots \dots \dots (1)$$

$$\because \mu = 1 \text{ (for air)} \quad \dots \dots \dots$$

Similarly the diameter of $(n + p)^{th}$ ring is given by

$$D_{n+p}^2 = 4(n + p)R\lambda \quad \dots \dots \dots (2)$$

So by equation (1) & (2)

$$\begin{aligned} D_{n+p}^2 - D_n^2 &= 4(n + p)R\lambda - 4nR\lambda \\ D_{n+p}^2 - D_n^2 &= 4nR\lambda + 4pR\lambda - 4nR\lambda \\ D_{n+p}^2 - D_n^2 &= 4pR\lambda \\ \frac{D_{n+p}^2 - D_n^2}{4pR} &= \lambda \\ \lambda &= \frac{D_{n+p}^2 - D_n^2}{4pR} \end{aligned}$$

2. Determination of refractive index of any liquid:

For air film $(D_{n+p}^2 - D_n^2)_{air} = 4pR\lambda \quad \because \mu = 1 \text{ for air} \quad \dots \dots \dots (1)$

In liquid $(D_{n+p}^2 - D_n^2)_{liquid} = \frac{4pR\lambda}{\mu} \quad \dots \dots \dots (2) \dots \dots \dots$

By equation (1) and (2)

$$\begin{aligned} \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{liquid}} &= \frac{4pR\lambda}{\left(\frac{4pR\lambda}{\mu}\right)} \\ \mu &= \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{liquid}} \end{aligned}$$

Diffraction:

1. Bending of the light from the sharp edges of the obstacle is called the diffraction.
2. The intensity of light outside the geometrical shadow of an obstacle and presence of light within its geometrical shadow is called the diffraction of light.
3. The deviation of light from the rectilinear path is called the diffraction.

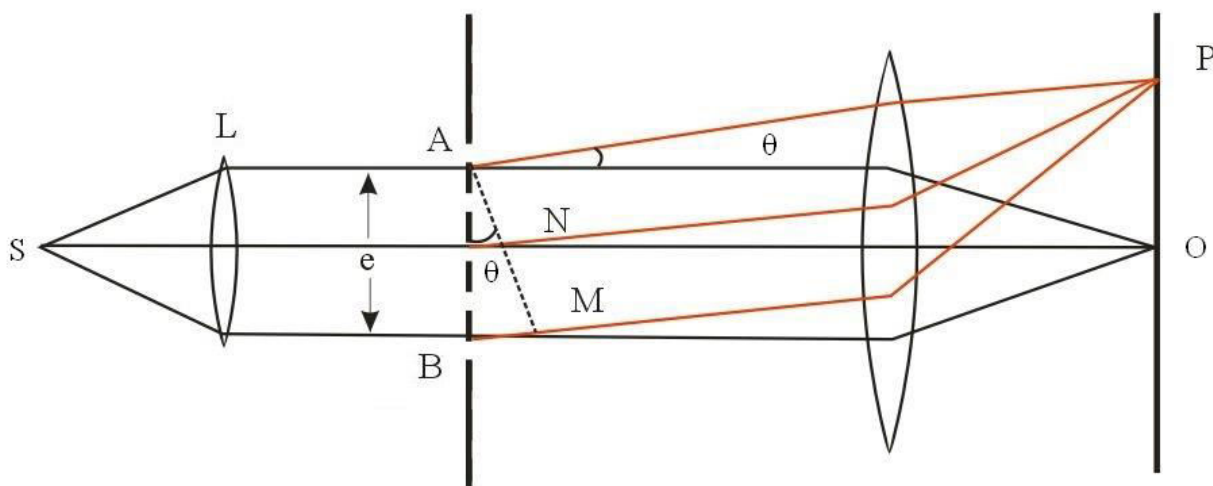
S.N.	Fresnel's diffraction	S.N.	Fraunhofer's Diffraction
1.	Either the source of light or screen or both are at finite distance from obstacle or aperture.	1.	Both the screen and source are effectively at infinite distance from the obstacle or aperture.
2.	Wavefront may be of any type i.e. plane, spherical or cylindrical.	2.	The incident wavefront is always a plane wavefront.
3.	No need to use the lenses.	3.	Lenses are required.
4.	Diffraction pattern is the image of obstacle or aperture.	4.	Diffraction pattern is the image of the source.
5.	Intensity of light at any point is found by the half period zone method which is not accurate.	5.	Intensity at any point is measured by the mathematical treatment which is more accurate method.

Difference between diffraction and interference:

S.N.	Interference		Diffraction
1.	This phenomenon is the result of interaction taking place between two separate wave front originating from two coherent sources.	1.	This phenomenon is the result of interaction of light between the secondary wavelengths originating from different points of the same wavefronts.
2.	The regions of minimum intensity are usually almost perfectly dark.	2.	The regions of minimum intensity are not completely dark.
3.	Interference fringes may or may not be of same width.	3.	Diffraction fringes are not of the same width.
4.	All maxima are of same intensity.	4.	The maxima are of varying intensities.

Fraunhofer's diffraction at a single slit:

Let parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit $AB = e$. According to Huygens's theory a plane wave front is incident normally on the slit AB . Each point of AB sends out secondary wavelets in all directions. The rays proceeding in the same direction as the incident rays are focused on O , while those diffracted through an angle θ are focused at P .



Figure(27): Fraunhofer's diffraction

To find the intensity at point P , we drop a normal ANM on the ray BP , the optical path from each point of the plane ANM to point P will be equal.

Now the path difference between the wavelets reaching the point P from point A and B is

$$\Delta = BM$$

But from $\triangle ABM$

$$\frac{BM}{AB} = \sin \theta$$

$$BM = AB \sin \theta$$

$$BM = e \sin \theta$$

$$\Delta = e \sin \theta$$

$$\text{so } \text{phase diff} = \frac{2\pi}{\lambda} \times \Delta$$

$$\text{phase diff} = \frac{2\pi}{\lambda} \times e \sin \theta \quad \dots \dots (1).$$

Now if we consider n number of infinite point sources of secondary wavelengths on the plane wave front AB then this can be divided into n equal parts, so phase difference between the waves obtained at the point P from any two consecutive parts

$$\phi = \frac{1}{n} \times \frac{2\pi}{\lambda} \cdot e \sin \theta \quad \dots \dots \dots$$

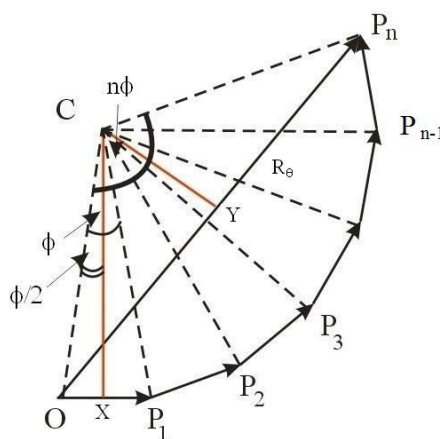
Now to find the intensity at point P there are following two methods are available

1. Phase diagram Method.
2. Integral Method.

Phase diagram method:

In the figure, draw vectors $OP_1, P_1P_2, P_2P_3 \dots \dots \dots$ such that the magnitude of each vector is A and angle between the two consecutive vectors is ϕ . The vector OP_n gives the resultant vector. Let the magnitude of the resultant vector OP_n is R_θ . If C is the centre of the polygon formed by the vector then by the simple geometry we can see that each vector subtends ϕ at the centre C and the angle subtended by the resultant vector OP_n at the centre is $n\phi$.

Let CX and CY are the normal drawn of first vector OP_1 and resultant vector OP_n from centre C .



Figure(28): Phase diagram

from right angle triangle ΔCXO

$$\frac{OX}{OC} = \sin \left(\frac{\phi}{2} \right)$$

$$OX = OC \sin \frac{\phi}{2}$$

But

$$OX = \frac{1}{2} OP_1 = \frac{A}{2}$$

$$\frac{A}{2} = OC \sin \frac{\phi}{2}$$

$\therefore OP = \frac{A}{\sin \frac{n\phi}{2}}$

... .. (3)

Similarly from ΔCOY

$$\frac{OY}{OC} = \sin \left(\frac{n\phi}{2} \right)$$

$$OY = OC \sin \frac{n\phi}{2}$$

But $OY = \frac{R_\theta}{2}$ so

$$\frac{R_\theta}{2} = OC \sin \frac{n\phi}{2}$$

... ..

By (3) and (4)

$$\frac{\left(\frac{R_\theta}{2}\right)}{\left(\frac{A}{2}\right)} = \frac{OC \sin \frac{n\phi}{2}}{OC \sin \frac{\phi}{2}}$$

$$R_\theta = A \frac{\sin \frac{n\phi}{2}}{\sin \frac{\phi}{2}} \quad \dots \dots (5)$$

Now putting the value of ϕ from the equation no. (2) we get

$$R_\theta = A \frac{\sin \left(\frac{n(2\pi e)}{2(n\lambda)} \sin \theta \right)}{\sin \left(\frac{1(2\pi e)}{2(n\lambda)} \sin \theta \right)} \quad \dots \dots (6)$$

Let $\frac{\pi e \sin \theta}{\lambda} = p$

Then $R_\theta = A \frac{\sin(p)}{\sin \frac{p}{n}}$

Now $\frac{p}{n}$ is very small so $\sin \frac{p}{n} \approx \frac{p}{n}$

Then $R_\theta = A \frac{\sin(p)}{\left(\frac{p}{n}\right)}$

So $R_\theta = nA \frac{\sin(p)}{p}$

$$R_\theta = R_0 \frac{\sin(p)}{p}$$

Where $R_0 = nA$

Now the intensity

$$I \propto R_\theta^2$$

$$I = kR_\theta^2$$

[Where k is a constant]

$$I = k \left[\frac{\sin(p)}{p} \right]^2$$

$$I = kR_0^2 \left[\frac{\sin(p)}{p} \right]^2$$

$$I = I_0 \left[\frac{\sin(p)}{p} \right]^2 \dots \dots \dots (7)$$

Where $I_0 = kR_0^2$

Conditions for maxima and minima:

From the equation $I = I_0 \left[\frac{\sin(p)}{p} \right]^2$ it is clear that the resultant intensity I at point P on the screen depends on the angle of diffraction θ or on P . For maxima, the derivation of I with respect to P must be zero. i. e.

$$\frac{d}{dp} \left\{ I_0 \left[\frac{\sin(p)}{p} \right]^2 \right\} = 0$$

$$I_0 \cdot 2 \cdot \frac{\sin(p)}{p} \cdot \left(-\frac{\cos(p)}{p^2} \right) = 0 \quad \dots\dots(8)$$

Condition for Minima:

For the minima, the first term in the above equation (8) should be zero i.e.

$$\frac{\sin(p)}{p} = 0$$

$$\sin(p) = 0$$

$$p = \pm m\pi$$

Putting the value of p we get-

$$\frac{\pi e \sin(p)}{\lambda} = \pm m\pi$$

$$e \sin(p) = \pm m\lambda$$

Where $m = 0, 1, 2, 3, \dots \dots \dots$

Condition for maxima:

Now the second term of equation (8) will show the condition for maxima-

$$\text{i.e.} \quad \frac{p \cos(p)}{p} = 0$$

$$p \cos(p) = \sin(p)$$

$$p = \frac{\sin(p)}{\cos(p)}$$

$$p = \tan(p)$$

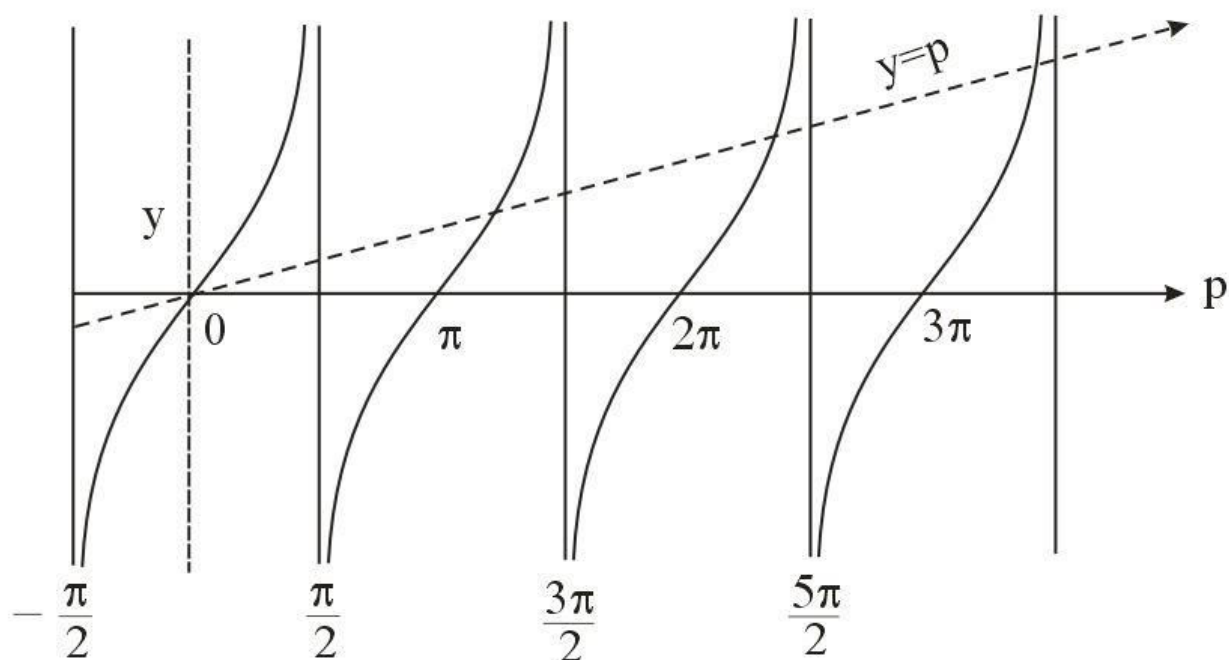
The condition for maxima is $p = \tan(p)$

To find the value of p for which the above condition may hold, we draw two curves

$$y = p \quad \dots \dots (9)$$

$$y = \tan(p) \quad \dots \dots (10)$$

On the same graph as shown



Figure(29): Graph

The value of p at the points of intersection of these two curves satisfy the equation $p = \tan p$

At the central maxima: $\theta = 0 \Rightarrow p = 0$

$$\text{So } \frac{\pi e \sin}{\lambda} = 0$$

So, the intensity at the principle maxima

$$I = I_0 \left(\frac{\sin p}{p} \right)^2$$

Applying the limits we get

$$\lim_{p \rightarrow 0} \left(\frac{\sin p}{p} \right)^2 = 1$$

$$\text{So } I = I_0 \quad (\text{maximum})$$

So at the principle maxima the intensity will be maxima.

Intensity for subsidiary maxima:

For subsidiary maxima the value of $\sin p$ must be maximum, for this the value of $p = \frac{\pi e \sin \theta}{\lambda}$ i.e. the value of $\sin \theta$ must be maximum i. e. $\sin \theta = 1$

$$\theta = (2m + 1) \frac{\pi}{2}$$

$$\text{i. e. } \theta = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \dots \dots$$

so at the,

first subsidiary maxima $I_1 = I_0 \left[\frac{\sin \frac{3\pi}{2}}{\frac{3\pi}{2}} \right]^2 = \frac{I_0}{22}$

Second subsidiary maxima $I_2 = I_0 \left[\frac{\sin \frac{5\pi}{2}}{\frac{5\pi}{2}} \right]^2 = \frac{I_0}{62}$

Third subsidiary maxima $I_3 = I_0 \left[\frac{\sin \frac{7\pi}{2}}{\frac{7\pi}{2}} \right]^2 = \frac{I_0}{120}$

Fraunhofer's diffraction at a double slit:

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon two parallel slit AB and

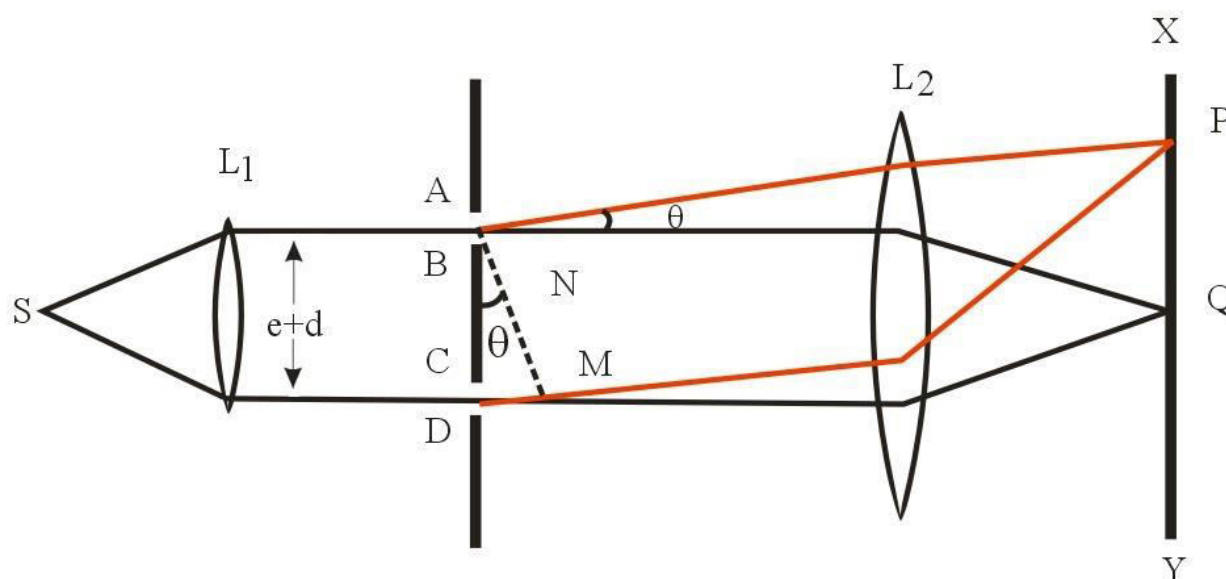


Figure (30): diffraction at a double slit.

Suppose each slit diffracts the beam in a direction making an angle θ with the direction of the incident beam.

From the theory of diffraction at a single slit the resultant amplitude is

$$R_{\theta} = R_0 \frac{\sin(p)}{p}$$

Where $p = \frac{\pi e \sin \theta}{\lambda}$ and R_0 is a constant

These two slits can be considered as two coherent source placed at the centre of the slits. Then resultant intensity at point P will be the result of interference between these two waves of same amplitude and phase difference ϕ

Now the resultant amplitude at point P

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \theta} \quad (\text{interference})$$

So

$$R = \sqrt{R_{\theta}^2 + R_{\theta}^2 + 2R_{\theta}R_{\theta} \cos \phi}$$

$$R = \sqrt{2R_{\theta}^2(1 + \cos \phi)}$$

$$R = R_{\theta} \sqrt{2 \cos^2 \frac{\phi}{2}}$$

$$R = 2R_0 \cos \frac{\phi}{2}$$

But we know that $R_0 = R \frac{\sin(p)}{p}$

$$\text{So } R = 2R_0 \left(\frac{\sin(p)}{p} \right) \cos \frac{\phi}{2} \quad \dots \dots (1).$$

Therefore the resultant intensity at point P will be

$$I \propto R^2$$

$$I = kR^2$$

Where k is a constant

Putting the value of R from equation (1)

$$I = k \left(2R_0 \left(\frac{\sin(p)}{p} \right) \cos \frac{\phi}{2} \right)^2 \quad \dots \dots (2)$$

$$\text{Let } I_0 = 4kR_0^2$$

Hence resultant intensity

$$I = I_0 \left(\frac{\sin(p)}{p} \right)^2 \cos^2 \left(\frac{\phi}{2} \right) \quad \dots \dots (3)$$

Condition for Minima:

From the equation (3) it is clear that the intensity will be minimum when $\sin p = 0 \Rightarrow p = \pm m\pi$

Where $m = 1, 2, 3 \dots$ but $m \neq 0$

So putting the value of p we get

$$\Rightarrow \frac{\pi e \sin}{\lambda} = m\pi$$

$$e \sin = m\lambda$$

Where $m = 1, 2, 3, \dots$ but $m \neq 0$

($\because m = 0$ is the condition for the maxima)

Condition for maxima:

From the equation (3) it is clear that the intensity will be maximum when term $\left(\frac{\sin(p)}{p} \right)^2$ will be maximum.

$$\sin p = 1$$

$$\Rightarrow p = (2m - 1) \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi e \sin}{\lambda} = (2m - 1) \frac{\pi}{2}$$

$$\Rightarrow e \sin = (2m - 1) \frac{\lambda}{2}$$

Where $n = 0, 1, 2, 3, 4, \dots$

Missing order maxima:

The condition for the interference maxima is given as $\Delta = n\lambda$

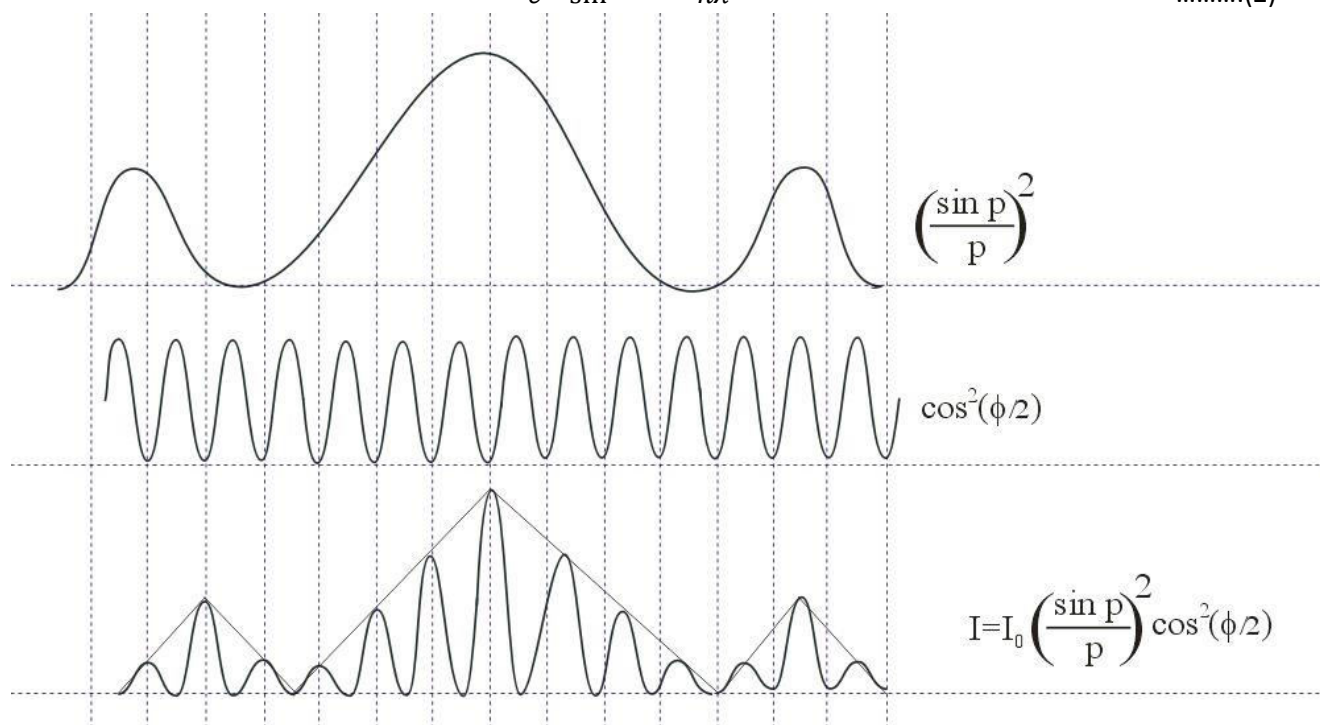
But $\Delta = e \sin \theta$ (from single slit)

So the condition for the interference maxima will be

$$(e + d) \sin \theta = m\lambda \quad \text{.....(1)}$$

And the condition for the diffraction minima is given as

$$e \sin \theta = n\lambda \quad \text{.....(2)}$$



Figure(31): Intensity graph of double slit diffraction

For certain value of d certain interference maxima become absent from the pattern. Let for some value of θ the following two conditions be satisfied simultaneously

Dividing the equation (1) by (2)

$$\frac{e + d}{e} = \frac{n}{m}$$

Case I: If $d = e$

$$\text{Then } \frac{e + d}{e} = \frac{n}{m}$$

$$\Rightarrow \frac{n}{m} = 2$$

$$\Rightarrow n = 2m$$

If $m = 1, 2, 3, \dots \Rightarrow n = 2, 4, 6, \dots$

This means that 2, 4, 6 etc order of interference maxima will be missed.

If $d = 3e$

Case I: If $d = 2e$

$$\begin{aligned} \text{Then } \frac{e + \lambda}{e} &= \frac{n}{m} \\ 3 &= \frac{n}{m} \\ \Rightarrow n &= 3m \end{aligned}$$

If $m = 1, 2, 3$, then $n = \dots 3, 6, 9 \dots$

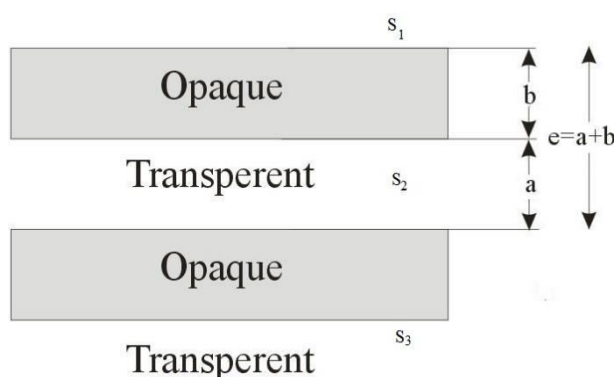
This means that 3, 6, 9 etc. will be missed.

Fraunhofer's Diffraction of N Parallel slit:

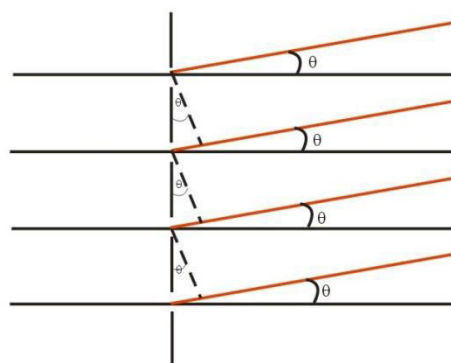
Diffraction Grating:

It is an arrangement consisting of several parallel and equidistant slits each of equal width.

It is constructed by drawing the several equidistance parallel lines on an optically plane glass plate with a pointed diamond. The distance between two consecutive slits is $a + b$ which is called the grating element. Generally the value of e for the grating to be used with the visual light is of the order of 10^{-6} m (i.e. 10000 lines drawn on 1 cm length of the grating).



Figure(32): Diffraction Grating

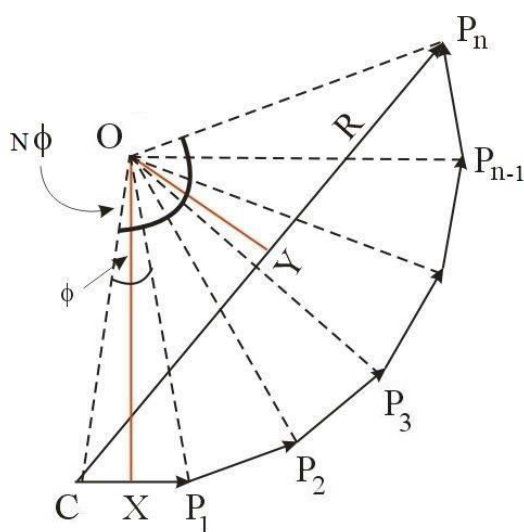


Figure(33): Diffraction Grating

Theory:

In figure, AB is a grating of N parallel and equidistance slits S_1, S_2, S_3, \dots . the width of each slit is ' a ' and width of opaque space between the two consecutive slits is ' b '. i.e. the grating element $e = a + b$

Let a plane wavefront of wavelength λ is incident normally on the grating. Then diffracted by it is focused on a screen by means of a convergent lens L on screen.



Figure(34): Phase Diagram

Intensity distribution: It is clear from the figure that diffracted waves do not reach a point on the screen in the same phase since their optical paths are not equal. The path difference between the two consecutive wave is $\Delta = e \sin \theta$

Therefore the phase diff.

$$\phi = \frac{2\pi}{\lambda} \times \Delta$$

Wave diffracted at an angle θ from each slit is $R_\theta =$

$$R_0 \left(\frac{\sin p}{p} \right) \quad \text{where} \quad p = \frac{\pi e \sin \theta}{\lambda}$$

[by single slits diffraction]

Now we can find the resultant amplitude due to the superposition of such N waves by phase diagram method. In figure we draw vector $OP_1, P_1P_2, P_2P_3 \dots$ such that magnitude of each vector is R_θ and the angle between the consecutive vector is ϕ . The vector OP_n which joints the initial points of first vector and final point of last vector is R and this vector sustained an angle $N\phi$. OX and OY are the normal plotted from the centre of polygon on first and resultant vectors.

From the figure in ΔCXO

$$\frac{OX}{OC} = \sin \frac{\phi}{2}$$

$$\Rightarrow OX = OC \sin \frac{\phi}{2}$$

$$\text{But } OX = \frac{1}{2} OP_1 = \frac{R_\theta}{2}$$

$$\therefore \frac{R_\theta}{2} = OC \sin \frac{\phi}{2} \quad \dots \dots \dots$$

Similarly in ΔCYO

$$\Rightarrow \frac{OY}{OC} = \sin \left(\frac{N\phi}{2} \right)$$

$$\Rightarrow OY = OC \sin \left(\frac{N\phi}{2} \right)$$

$$\text{But } OY = \frac{OP_n}{2} = \frac{R}{2}$$

$$\therefore \frac{R}{2} = OC \sin \left(\frac{N\phi}{2} \right) \quad \dots \dots \dots (2)$$

Dividing equation (2) by (1)

$$\frac{\left(\frac{R}{2} \right)}{\left(\frac{R_\theta}{2} \right)} = \frac{OC \sin \left(\frac{N\phi}{2} \right)}{OC \sin \left(\frac{\phi}{2} \right)}$$

$$\frac{R}{R_\theta} = \left[\frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right]$$

$$R = R_\theta \left[\frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right]$$

On substituting the value of R_θ we get

$$R = R_o \left(\frac{\sin p}{p} \right) \left[\frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right]$$

Multiplying and dividing by N we get

$$R = NR_0 \left(\frac{\sin \frac{\phi}{p}}{p} \right)^2 \left[\frac{\sin \frac{N\phi}{2}}{N \sin \frac{\phi}{2}} \right]^2$$

So the resultant intensity will be given as

$$I \propto R^2$$

$$I = kR^2$$

Where k is the proportionality constant

Putting the value of R from (3) we get

$$I = k N^2 R_0^2 \left(\frac{\sin \frac{\phi}{p}}{p} \right)^2 \left[\frac{\sin \frac{N\phi}{2}}{N \sin \frac{\phi}{2}} \right]^2$$

$$I = I_0 \left(\frac{\sin \frac{\phi}{p}}{p} \right)^2 \left[\frac{\sin \frac{N\phi}{2}}{N \sin \frac{\phi}{2}} \right]^2 \quad \dots \dots (3).$$

$$\text{Where } I_0 = k N^2 R_0^2$$

In this expression the term $\left(\frac{\sin \frac{\phi}{p}}{p} \right)^2$ represents the intensity due to diffraction due to a single slit, while second

term $\left[\frac{\sin \left(\frac{N\phi}{2} \right)}{N \sin \frac{\phi}{2}} \right]^2$ represents the intensity due to interference of wave obtained from N slits.

Condition for Principle Maxima:

For principal maxima the path difference will be zero so the phase diff then $\sin \frac{\phi}{\lambda} = 0 \Rightarrow \pm n\pi$ where

$n = 0, 1, 2$ then $\sin \frac{N\phi}{2}$ is also zero and in the limit when $\sin \frac{\phi}{\lambda} \rightarrow 0$ the value of term $\left[\frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right]^2$ will be

N . Hence from equation the resultant intensity will be maximum.

i.e.

$$\lim_{\frac{\phi}{2} \rightarrow 0} \left[\frac{\sin \frac{N\phi}{2}}{N \sin \frac{\phi}{2}} \right] = 1$$

So we have

$$I = I_0 \left(\frac{\sin \frac{\phi}{p}}{p} \right)^2 \quad \dots \dots (4)$$

Which is the intensity at principle maxima i.e. similar to the intensity by a single slit.

Condition for Minima:

From the equation (3) it is clear that the intensity will be minimum when $\sin \frac{N\phi}{2} = 0$ but $\sin \frac{\phi}{2} \neq 0$

$$\text{i.e.} \quad \sin \frac{N\phi}{2} = 0$$

$$\Rightarrow \quad \frac{N\phi}{2} = \pm m\pi$$

Where $m = 1, 2, 3, \dots$

$$\text{But } \phi = \frac{2\pi e \sin \theta}{\lambda}$$

$$\text{So } \frac{N}{2} \frac{2\pi e \sin \theta}{\lambda} = m\pi \quad \dots \dots (5).$$

$$e \sin \theta = \frac{m}{N} \lambda$$

This is the condition for the minimum intensity for N-slit diffraction.

Condition for Secondary maxima:

$$\text{Condition for maxima is } \frac{dI}{d\phi} = 0$$

So from equation (3)

$$\frac{d}{d\phi} \left[I_0 \left(\frac{\sin \phi}{p} \right)^2 \left\{ \frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right\}^2 \right] = 0$$

$$\frac{I_0}{N^2} \left(\frac{\sin \phi}{p} \right)^2 \frac{d}{d\phi} \left\{ \frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right\}^2 = 0$$

$$\frac{I_0}{N^2} \left(\frac{\sin \phi}{p} \right)^2 \left[\frac{2 \sin \left(\frac{N\phi}{2} \right) \cos \left(\frac{N\phi}{2} \right) \cdot \frac{1}{2} \cos \left(\frac{\phi}{2} \right) - \sin \left(\frac{N\phi}{2} \right) \left(\frac{1}{2} \cos \left(\frac{\phi}{2} \right) \right)}{\left\{ \sin \left(\frac{\phi}{2} \right) \right\}^2} \right] = 0$$

$$\frac{N}{2} \sin \left(\frac{\phi}{2} \right) \cdot \cos \left(\frac{N\phi}{2} \right) - \frac{1}{2} \sin \left(\frac{N\phi}{2} \right) \cdot \cos \left(\frac{\phi}{2} \right) = 0$$

$$\frac{N}{2} \sin \left(\frac{\phi}{2} \right) \cdot \cos \left(\frac{N\phi}{2} \right) = \frac{1}{2} \sin \left(\frac{N\phi}{2} \right) \cdot \cos \left(\frac{\phi}{2} \right) \quad \dots \dots (6)$$

$$\Rightarrow N \tan \left(\frac{\phi}{2} \right) = \tan \left(\frac{N\phi}{2} \right) \quad \dots \dots \dots$$

again by equation (6)

$$N \tan \left(\frac{\phi}{2} \right) \cos \left(\frac{N\phi}{2} \right) = \sin \left(\frac{N\phi}{2} \right)$$

$$\Rightarrow \sin \left(\frac{N\phi}{2} \right) = N \tan \left(\frac{\phi}{2} \right) \cos \left(\frac{N\phi}{2} \right)$$

$$\sin \left(\frac{N\phi}{2} \right) = \frac{N \tan \left(\frac{\phi}{2} \right)}{\sec \left(\frac{N\phi}{2} \right)}$$

$$\sin \left(\frac{N\phi}{2} \right) = \frac{N \tan \left(\frac{\phi}{2} \right)}{\sqrt{1 + \tan^2 \left(\frac{N\phi}{2} \right)}}$$

$$\sin \left(\frac{N\phi}{2} \right) = \frac{N \tan \left(\frac{\phi}{2} \right)}{\sqrt{1 + \tan^2 \left(\frac{\phi}{2} \right)}} \quad \text{By (7)}$$

On squaring both sides we get

$$\begin{aligned} \sin^2 \left(\frac{N\phi}{2} \right) &= \frac{N^2 \left(\frac{\sin \left(\frac{\phi}{2} \right)}{\cos \left(\frac{\phi}{2} \right)} \right)^2}{1 + \tan^2 \left(\frac{\phi}{2} \right)} \\ \frac{\sin^2 \left(\frac{N\phi}{2} \right)}{\sin^2 \left(\frac{\phi}{2} \right)} &= \frac{N^2}{\left[1 + \tan^2 \left(\frac{\phi}{2} \right) \right] \cdot \cos^2 \left(\frac{\phi}{2} \right)} \\ \frac{\sin^2 \left(\frac{N\phi}{2} \right)}{\sin^2 \left(\frac{\phi}{2} \right)} &= \frac{N^2}{\left[\cos^2 \left(\frac{\phi}{2} \right) + \tan^2 \left(\frac{\phi}{2} \right) \cdot \cos^2 \left(\frac{\phi}{2} \right) \right]} \\ \frac{\sin^2 \left(\frac{N\phi}{2} \right)}{\sin^2 \left(\frac{\phi}{2} \right)} &= \frac{N^2}{\cos^2 \left(\frac{\phi}{2} \right) + \frac{\sin^2 \left(\frac{\phi}{2} \right)}{\cos^2 \left(\frac{\phi}{2} \right)} \cdot \cos^2 \left(\frac{\phi}{2} \right)} \\ \frac{\sin^2 \left(\frac{N\phi}{2} \right)}{\sin^2 \left(\frac{\phi}{2} \right)} &= \frac{N^2}{\left[\cos^2 \left(\frac{\phi}{2} \right) + \sin^2 \left(\frac{\phi}{2} \right) \right]} \\ \frac{\sin^2 \left(\frac{N\phi}{2} \right)}{\sin^2 \left(\frac{\phi}{2} \right)} &= \frac{N^2}{1 - \sin^2 \left(\frac{\phi}{2} \right) + \sin^2 \left(\frac{\phi}{2} \right)} \\ \left[\frac{\sin \left(\frac{N\phi}{2} \right)}{\sin \left(\frac{\phi}{2} \right)} \right]^2 &= \frac{N^2}{1 + (N-1) \sin^2 \left(\frac{\phi}{2} \right)} \\ \left[\frac{\sin \left(\frac{N\phi}{2} \right)}{N \sin \left(\frac{\phi}{2} \right)} \right]^2 &= \frac{1}{1 + (N-1) \sin^2 \left(\frac{\phi}{2} \right)} \quad \text{..... (7)} \end{aligned}$$

So by equation (3) and (7)

$$I = I_0 \left(\frac{\sin p}{p} \right)^2 \times \frac{1}{1 + (N-1) \sin^2 \left(\frac{\phi}{2} \right)}$$

This is the expression for the intensity at the subsidiary maxima in N-slit diffraction.

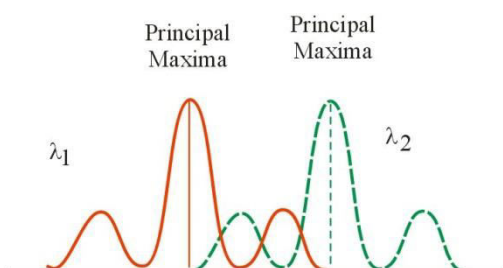
Resolving Power of Optical Instrument:

To distinguish two close object is called geometrical resolution and the ability of an optical instrument to distinguish the image of very close object is called the resolving power of that optical instrument.

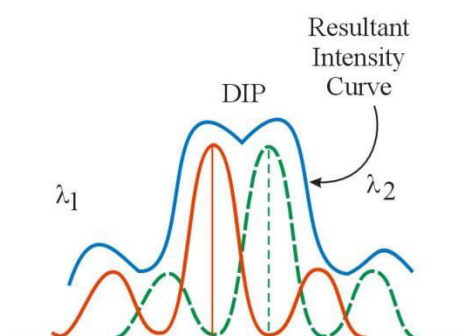
The ability of instrument to produce the separate diffraction pattern is known as resolving power.

Raleigh's criterion of resolution:

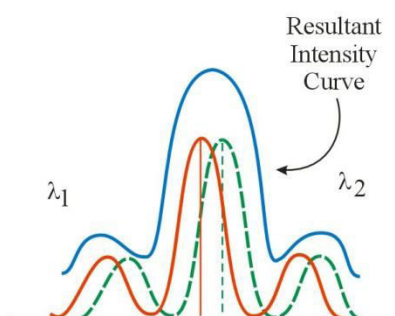
According to this criterion two sources are resolved by an optical instrument when the central maxima in the diffraction pattern is fall over the first minima in the diffraction pattern of the second maxima and vice versa.



Figure(35): Two separate maxima



Figure(36): Condition of just resolution



Figure(37): Maxima that cannot be resolve

In order to illustrate the criterion let us consider the resolution of two wavelengths λ_1 and λ_2 . Figure shows the intensity curve of the diffraction pattern of two wavelengths. The diffraction in wavelength is such that their principal maxima are separately visible. There is a distinct point of zero intensity in between the two. Hence the two wavelengths are resolved.

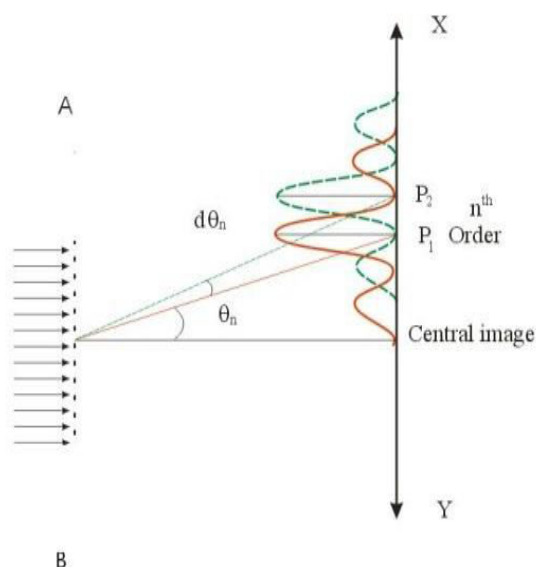
In the case when there is small dip between the maxima of λ_1 and λ_2 such that the central maxima of wavelength λ_1 coincide with the first minima of λ_2 and vice versa as shown in the figure (36). The resultant intensity curve has a dip in the middle of the two central maxima. Thus two wavelengths can be distinguished from one another

If the difference between the two wavelength λ_1 and λ_2 is so small that the maxima corresponding to wavelength come still closer as shown in the figure (37) the resultant intensity curve in this case is quit smooth without any dip, thus wavelengths cannot be resolved.

Resolving power of Grating:

The resolving power of a diffraction grating is defined as the capacity to form separate diffraction maxima of

two wavelengths without which they are very close to each other. This is measured by $\frac{\lambda}{d\lambda}$.



Figure(38): Formation of diffraction pattern by a grating

Let AB is a plane diffraction grating having grating element $e = (a \text{ and } b)$ total numbers of number of slits. Let a beam of light having two wavelengths λ and $\lambda + d\lambda$ normally incident on the grating. P_1 is the n^{th} primary maxima of spectral line of wavelength λ at an angle of diffraction θ_n and P_2 is the n^{th} primary maxima of wavelength $(\lambda + d\lambda)$ diffraction angle $(\theta_n + d\theta)$

The principal maxima of λ in θ_n direction will be

$$(a + b) \sin \theta = n\lambda \dots \dots \dots (1)$$

And the equation of minima $N(a + b) \sin \theta = m\lambda$

Where m is an integer except 0, N , $2N$... , .. because for these values of m the condition for maxima is satisfies and we obtain diffraction maxima.

Now first maxima adjacent to n^{th} principle maxima

$$(a + b) \sin(\theta_n + d\theta) = n(\lambda + d\lambda) \dots \dots \dots (2)$$

And first minima

$$N(a + b) \sin(\theta_n + d\theta) = (nN + 1)\lambda \dots \dots \dots (3)$$

Now multiplying the equation (2) by N we have

$$N(a + b) \sin(\theta_n + d\theta) = nN(\lambda + d\lambda) \dots \dots \dots (4)$$

By (3)& (4)

$$(nN + 1)\lambda = nN(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nNd\lambda$$

$$\frac{\lambda}{d\lambda} = nN$$

