(b) Determine the discrete numeric function corresponding to the following generating function :

(i) $A(z) = \dfrac{7z^2}{(1-2z)(1+3z)}$

(ii) $A(z) = \dfrac{1}{(5-6z+z^2)}$

10. (a) Find the generating function of the sequence y_0, y_1, y_2 ..... defined as follows :

$$y_0 = 0, y_1 = 1$$

$$y_n + 2y_{n-1} - 15y_{n-2} = 0, \text{ for } n \geq 2$$

(b) Determine the particular solution for the difference equation :

$$a_r - 3a_{r-1} + 2a_{r-2} = 2^r$$

Total No. of Questions : 10] [Total No. of Printed Pages : 4

Roll No. ....

MCA – 102

M. C. A. (First Semester) EXAMINATION, Dec., 2011

(Grading/Non-Grading)

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

(MCA – 102)

Time : Three Hours

Maximum Marks : GS : 70, NGS : 100

Note : Attempt *one* question from each Unit. All questions carry equal marks.

Unit – I

1. (a) It is known that at the university 60 percent of the professors play tennis, 50% of them play bridge, 70% play jog, 20% play tennis and bridge, 30% play tennis and jog and 40% play bridge and jog. If someone claimed that 20% of the professors play jog, bridge and tennis, would you believe this claim. Why ?

(b) If R is an equivalence relation on a set A, then prove that R^{-1} is an equivalence relation on the set A.

2. (a) If $f : X \to Y$ and $g : Y \to Z$ be two one-one onto maps, then prove that $g \circ f : X \to Z$ is also one-one onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(b) Prove by mathematical induction that $2^n \geq n^2$ for $n \geq 4$.

Unit–II

3. (a) Prove that the following statement is a contradiction :

$$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$$

(b) Prove that the following is tautology :

$$(p \wedge q \Rightarrow r) \Leftrightarrow (p \Rightarrow r) \vee (q \Rightarrow r)$$

4. (a) Let a, b, c be elements in a lattice $(A, \leq)$, then prove that :

(i) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(ii) $(a \wedge b) \vee (a \wedge c) \leq a \wedge (b \vee c)$

(b) Show that a lattice is distributive if and only if for any elements a, b, c in the lattice :

$$(a \vee b) \wedge c \leq a \vee (b \wedge c)$$

Unit–III

5. (a) Let H be a subgroup of G, prove that :

(i) $H = Ha$ if and only if $a \in H$

(ii) $Ha = Hb$ if and only if $ab^{-1} \in H$

(b) Let G be the set of the non-zero real numbers and let $a * b = ab/2$, then show that $(G, *)$ is an abelian group.

6. (a) State and prove Lagrange's theorem for a finite group.

(b) If the system $(R, +, \cdot)$ be a ring R, then prove that :

(i) $a \cdot (-b) = (-a) \cdot b = -(a \cdot b), \forall a, b \in R$

(ii) $(-a) \cdot (-b) = a \cdot b, \forall a, b \in R$

(iii) $a \cdot (b - c) = a \cdot b - a \cdot c, \forall a, b, c \in R$

(iv) $a \cdot 0 = 0 \cdot a = 0, \forall a \in R$

Unit–IV

7. (a) Prove that the maximum number of edges in a simple garph with n vertices and k components is :

$$(n - k)(n - k + 1)/2$$

(b) Determine a minimum spanning tree for a graph shown below :

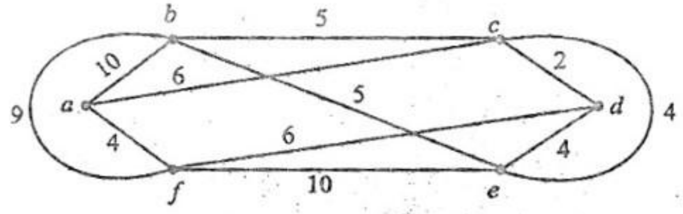

Fig. 1

8. (a) Determine a shortest path between a and z in the following graph :

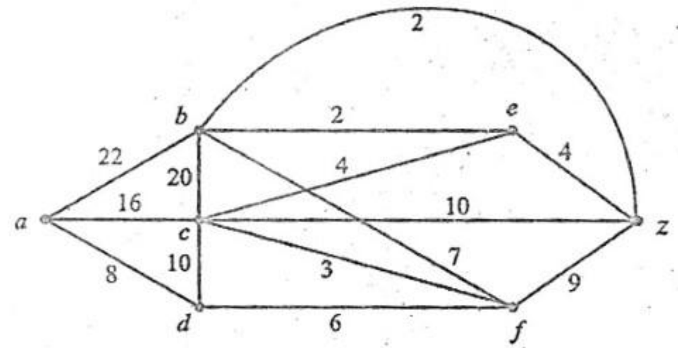

Fig. 2

(b) A graph is given by the following adjacency matrix, check whether it is connected or not ?

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

Unit–V

9. (a) Solve the recurrence relation :

$$a_r - 5a_{r-1} + 6a_{r-2} = r(r-1) \text{ for } r \geq 2.$$