Total No. of Questions :5] *[Total No. of Printed Pages : 4*

Roll No .....

MCA - 102

MCA. I Semester

Examination, June 2016

Mathematical Foundation of Computer Science

Time : Three Hours

Maximum Marks : 70

Note: i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.

ii) All parts of each question are to be attempted at one place.

iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max. 400 words) carry 7 marks.

iv) Except numericals, Derivation, Design and Drawing etc.

Unit - I

1. a) If $A = \{x : 1 < x < 4, x \in I\}$ and $B = \{x : 2 < x < 7, x \in I\}$ then write $A \cap B$ and $A \cup B$.

   b) Explain the principle of inclusion and exclusion.

   c) Show that the relation $R = \{(a, b) : a, b \in I$ and $a-b$ is divisible by $3\}$ is an equivalence relation.

   d) Prove using the mathematical induction that $5^{2n}-1$ is divisible by 24, where n is any positive integer.

   OR

   If $f : X \to Y$ is one-one and onto, then prove that $f^{-1} : Y \to X$ is also one-one and onto.

Unit - II

2. a) Prove that $(p \wedge q) \to (p \vee q)$ is a tautology.

   b) Prove that the statements $(p \vee q)$ and $(\sim p \wedge \sim q)$ are contradictory to each other.

   c) Show that the relation $\subseteq$ is a partial order relation on the set of all integers.

   d) Prove that every chain is a distributive lattice.

   OR

   Express the function

   $f(x, y, z) = x.y' + x.z + x.y$

   into disjunctive normal form.

Unit - III

3. a) Define :

      i) Groupoid

      ii) Semi group

      iii) Monoid

      iv) Finite and infinite group

   b) Define :

      i) Sub-group

      ii) Sub-ring

   c) Show that the inverse of every element of a group is unique.

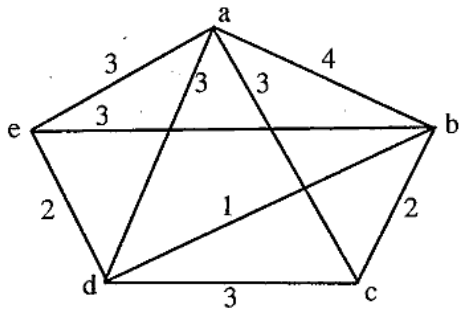
   d) Show that the set of numbers of the form $a+b\sqrt{2}$ with a and b as rational numbers is a field.

OR

Show that the set of four fourth roots of unity namely (1, –1, i, –i) forms an abelian group with respect to multiplication.

Unit - IV

4. a) Define with example:
   i) Complete graph
   ii) Bipartite graph
   b) Define:
   i) Tree
   ii) A spanning tree
   iii) Regular graph
   iv) Isolated vertex
   c) Show that every connected graph has at least one spanning tree.
   d) Find the minimum spanning tree for the weighted graph:



OR

Show that in any graph, the number of vertices of odd degree is always even.

Unit - V

5. a) Define sum and multiplication of two discrete numeric functions.
   b) Define generating function and write the generating function A(z) of the numeric function $a_r = 3^r$.
   c) Determine the discrete numeric function to the generating function

$$A(z) = \frac{2}{1-4z^2}$$

   d) Solve the recurrence relation

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2, r \geq 2$$

OR

Solve the recurrence relation :

$$a_r - 6a_{r-1} + 8a_{r-2} = 0, \text{ given } a_0 = 3, a_1 = 2.$$

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