

M. E./M. Tech. (First Semester)  
EXAMINATION, Feb./March, 2009

## ADVANCED COMPUTATIONAL MATHEMATICS

Time : Three Hours

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Maximum Marks : 100

Minimum Pass Marks : 40

**Note :** Attempt any five questions. All questions carry equal marks.

1. (a) Show that the mapping  $f: V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ , defined by  $f(a, b, c) = f(a, b)$  is Linear Transformation. What is the Kernel of this transformation ?
- (b) Define linearly dependent and independent sets. Check whether the vectors are  $(2, 1, 1)$ ,  $(0, 5, -1)$ ,  $(-1, 2, -1)$  are Linearly independent or Linearly dependent.
2. (a) Prove that Hermite Polynomial :

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2}).$$

- (b) Solve numerically :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

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With  $u(x, 0) = \sin \pi x$ ,  $0 \leq x \leq 1$ ;  $u(0, t) = u(1, t) = 0$  carry out computation for two levels taking  $h = \frac{1}{3}$ ,  $k = \frac{1}{36}$ .

3. (a) Solve by the method of separation of variable :

$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

given that  $u = 3e^{-y} - e^{-5y}$  when  $x = 0$ .

- (b) Solve by relaxation method, the Laplace equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Inside the square bounded by the lines  $x = 0, x = 4, y = 0, y = 4$  given that  $u = x^2 y^2$  on boundary.

4. (a) Six coins are tossed 6400 times using Poisson distribution, find the approximate probability of getting 6 heads  $x$  times and 2 times.
- (b) If the skulls are classified as A, B, C, according as the length breadth index as under 75, between 75 and 80, over 80, find approximately (assuming that distribution is normal) the mean and standard deviation of a series in which A are 58%, B are 38%, C are 4% being

given that  $f(t) = \frac{1}{2\pi} \int_0^t \frac{-x^2}{2} dx$ , then :

$$f(0.20) = 0.08, f(1.75) = 0.46.$$

5. (a) Explain the following with example :

- Hypothesis
- Testing of hypothesis
- Theory of estimation
- Discrete Random variable.

- (b) A coin is tossed until the head appears. What is expectation of the number of tosses ?

6. (a) Define Stochastic Process and Markov Process with example.

- (b) Test the following transition matrix to see if the Markov chain is regular and ergodic where  $x$  is some positive  $p_{ij}$  value :

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & x & x & 0 \\ x & 0 & 0 & x \\ x & 0 & 0 & x \\ 0 & x & x & 0 \end{bmatrix} \end{matrix}$$

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7. (a) Obtain the steady state solution of (M/M/1) : ( $\infty$ /FCFS) system and also find expected value of queue length  $n$ .
- (b) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call assumes to be distributed experimentally with mean 3 minutes. Then :
- What is the probability that a person arriving at the booth will have to wait ?
  - What is the average length of the queues that form time to time ?
8. (a) Use Rayleigh-Ritz method to solve the equation :

$$\frac{d^2 y}{dx^2} + y = x, y(0) = 0, y(1) = 1.$$

- (b) Use Galerkin's method to solve the equation :

$$\frac{d^2 y}{dx^2} - y + x = 0, y(0) = y(1) = 0.$$