

M. E./M. Tech. (First Semester)

EXAMINATION, June, 2012

(Grading/Non-Grading)

ADVANCED MATHEMATICS

(MMTP/MMCM/MMMD/MMIE/MMPD-101)

Time : Three Hours

Maximum Marks : $\begin{cases} GS : 70 \\ NGS : 100 \end{cases}$ **Note :** Attempt any five questions. All questions carry equal marks.

1. (a) If f is a mapping from $V_3(\mathbf{F})$ into $V_2(\mathbf{F})$, defined as $f(x, y, z) = (y, z)$, then show that f is linear.

- (b) Show that the set :

$$W = \{(a, b, c) : a - 3b + 4c = 0 \quad \forall a, b, c \in \mathbf{R}\}$$

is a sub-space of the 3-tuple vector space $V_3(\mathbf{R})$.

2. (a) Prove that :

$$H_n(-x) = (-1)^n H_n(x).$$

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- (b) Show that :

$$\int_0^\infty e^{-x^2 - 2ax} dx = \frac{\sqrt{\pi}}{2} e^{a^2} [1 - \operatorname{erf}(a)].$$

3. (a) Solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, subject to the condition :

$$u(x, 0) = 6e^{-3x}.$$

- (b) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in figure. Iterate until the maximum difference between two successive values at point is less than 0.001.

4. (a) Solve the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x \leq 5, \quad 0 \leq t \leq 1.25$$

subject to the conditions $u(0, t) = u(5, t) = 0, t > 0 :$

$$\left. \begin{aligned} u(x, 0) &= x^2(5-x) \\ u_t(x, 0) &= 0 \end{aligned} \right\} \quad 0 \leq x \leq 5.$$

- (b) Write notes on the following :

- (i) WFT
- (ii) Wavelet transform
- (iii) Haar transform.

5. (a) Two dice are thrown. Find the probability that the sum of the faces is :

- (i) 7 or 8
- (ii) more than 8.

- (b) In four tosses of a coin, let x be the number of heads. Calculate the expected value of x . What can you say about the expectation of number of tails ?

6. (a) Find the mean and variance of Poisson distribution.

- (b) The mean and variance of a Binomial distribution are 4 and $\frac{4}{3}$ respectively. Find :

- (i) the probability of 2 successes
- (ii) the probability of more than two successes
- (iii) the probability of more than three successes.

7. Derive the expression :

$$P_n(t) = P[x(t) = n] = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$

$$n = 0, 1, 2, \dots, \infty$$

in case of a Poisson process.

8. (a) What is the difference between FEM and DFT ?

- (b) Solve the boundary value problem :

$$y'' - y + x = 0, \quad (0 \leq x \leq 1), \quad y(0) = y(1) = 0$$

by Rayleigh-Ritz method.

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