June : 2016 (CBCS)

Note : **Max. marks : 60**

(i) Attempt any five questions

(ii) All questions carry equal marks.

(iii) Answer should be precise and to the point only

(iv) Assume suitable data if necessary and state them clearly.

Q.1 (a) Define the term "Force" and state clearly the effects of force. What are the various characteristics of a force?

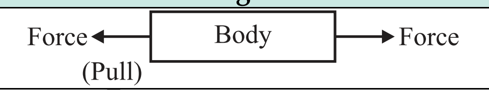
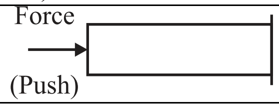
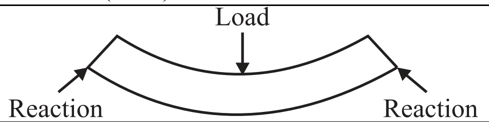
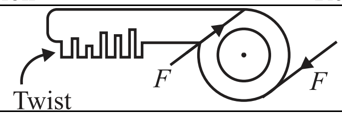
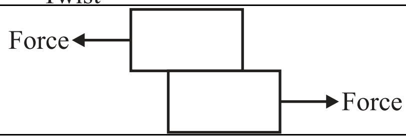
Ans. ***"Force is an external agent which can change the state of motion, shape and size of a body".***

It means that, on application of a force, a body at rest may start moving, or a moving body may come at rest.

A force can also deform a body i.e. it can bend, twist or break a body.

Force can give basically five effects, listed below in table :

Table : Effects of forces

S. No.	Force	Diagram	Effect
1.	**Pull**		**Elongation :** It may increase the length.
2.	**Push**		**Contraction :** It may decrease the length.
3.	**Bend**		**Bending :** It may bend the body.
4.	**Twist**		**Twisting :** It may twist the body.
5.	**Shear**		**Shear :** It may break the body.

From table given above we can conclude that, a same force can give five type of effects, if we only change its direction.

It simply means that force is a direction dependent quantity. Such type of quantities are known as vector quantity.

Value of force is given by Newton's second law of motion which states that, ***"The rate of change of momentum is directly proportional to the impulse of force".***

$$\vec{F} = m\vec{a}$$

SI unit : *Newton*, it is denoted by N. 1 N is a force which can produce an acceleration of 1m/s^2 in a body of mass 1 kg.

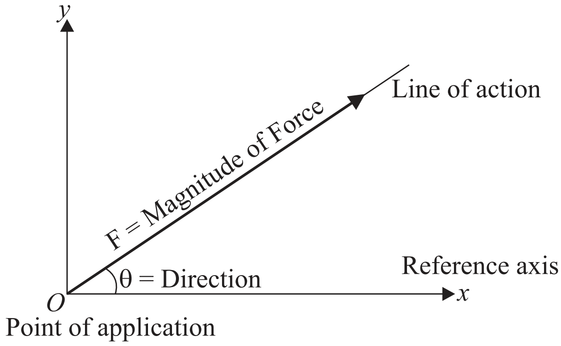
$$1\,\text{N} = \frac{1\,\text{kg} \cdot 1\,\text{m}}{\text{s}^2} \text{ or } 1\,\frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

CGS unit : *Dyne*, 1 dyne force is the force which can produce an acceleration of 1cm/s^2 in a body of mass 1 gram.

$$1\,\text{dyne} = 1\,\frac{\text{gm} \cdot \text{cm}}{\text{s}^2}$$

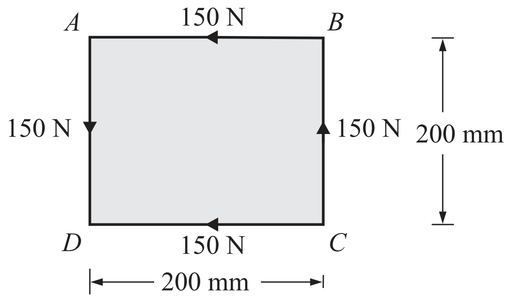
In order to define a force completely, the following characteristics should be mentioned :

1. **Magnitude of the force :** The magnitude of force is simply the value of force.
2. **Line of action :** It is the imaginary line along which force is acting.
3. **Direction of the force :** The angle through which line of action of force is inclined with the reference axis.
4. **Point of application :** The point at which the force acts is called point of application.



Q.1	**(b)**	A square ABCE has sides equal to 20 mm forces of 150 N each act along AB and CD and 250 N each along CB and AD. Find the moment of the couple. Which will keep the system in equilibrium.

Ans. **Given :**



Moment of couple *CB* and *AD* :

$$M_1 = 250 \times 0.2$$

$$M_1 = 50 \text{ Nm (Anti Clock Wise)}$$ **Ans.**

Moment of couple *AB* and *CD* :

$$M_2 = 150 \times 0.2$$

$$M_2 = 30 \text{ Nm (Clock Wise)}$$ **Ans.**

Total moment of couple which will bring the system in equilibrium :

$$M = M_1 - M_2$$

$$M = 50 - 30$$

$$M = -20\,\text{Nm} \text{ or } 20\,\text{Nm (Anti Clock Wise)}$$ **Ans.**

Q.2	**(a)**	State and prove parallelogram law of forces.

Ans. **Law of parallelogram of forces :** *If two force acting at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal passing through their point of intersection represents the resultant in both magnitude and direction.*

Proof :

Let the forces $\vec{P}$ and $\vec{Q}$ act at O. Let OA and OB represent the forces P and Q acting at an angle θ.
Complete the parallelogram $OACB$.
Draw CD perpendicular to OA.
Let $\angle COA = \alpha$, $OC = R$ denote the magnitude of the resultant and α is the direction of resultant.

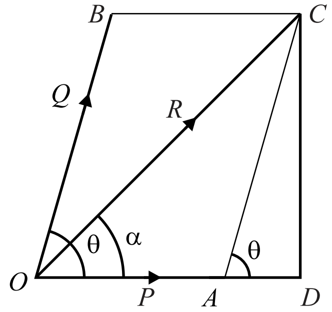


Fig.

From ΔADC,

$$\cos\theta = \frac{AD}{AC}$$

$$AD = AC\cos\theta$$

$$AD = Q\cos\theta$$

Now, $$OD = OA + AD = OA + AC\cos\theta$$

$$OD = P + Q\cos\theta \qquad [\because AC = OB = Q]$$

Also, $$DC = AC\sin\theta = Q\sin\theta$$

Resultant :

$$OC^2 = OD^2 + DC^2$$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

i.e. $$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \ldots\text{(i)}$$

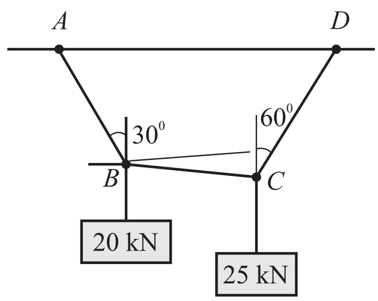
Direction of resultant :

$$\tan\alpha = \frac{CD}{OD} = \frac{Q\sin\theta}{P + Q\cos\theta} \qquad \ldots\text{(ii)}$$

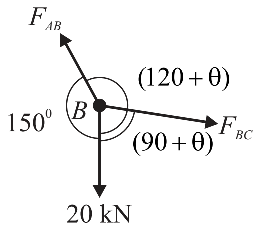
Equation (i) and equation (ii) give the required magnitude and direction of the resultant.

Q.2 **(b)** A wire is fixed at two points A and D at same level. Two weights 20 kN and 25 kN are suspended at B and C respectively. When equilibrium is reached it is found that inclination at AB is 30^0 and that of CD is 60^0 to vertical. Determine the tension in the segments AB. BC and CD of the rope and also the inclination of BC to the vertical.

Ans. **Given :**



FBD of B :



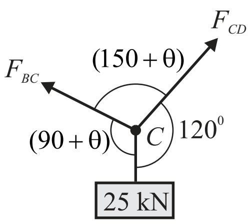
Applying lami's theorem at B,

$$\frac{F_{AB}}{\sin\left(90^0-\theta\right)}=\frac{F_{BC}}{\sin 150^0}=\frac{20}{\sin\left(120^0+\theta\right)}$$

$$F_{BC}=\frac{20\times\sin(150^0)}{\sin(120^0+\theta)} \quad \text{...(i)}$$

$$F_{AB}=\frac{20\times\sin(90^0-\theta)}{\sin(120^0+\theta)} \quad \text{...(ii)}$$

FBD at C :



Applying Lami's theorem at C,

$$\frac{F_{CD}}{\sin(90^0+\theta)}=\frac{F_{BC}}{\sin(120^0)}$$

$$F_{BC}=\frac{25\times\sin(120^0)}{\sin(150^0-\theta)} \quad \text{...(iii)}$$

$$F_{CD}=\frac{25\times\sin(90+\theta)}{\sin(150^0-\theta)} \quad \text{...(iv)}$$

From (i) and (iii), we get

$$\frac{20\sin(150^0)}{\sin(120^0+\theta)}=\frac{25\times\sin(120)}{\sin(150^0-\theta)}$$

On solving,

$$\theta=35.2087^0$$ **Ans.**

Q.3 **(a)** State and prove the Varignon's principle of moments.

Ans. French mathematician Varignon's gave the following theorem which is also known as principle of moments.

It states that : ***"The algebraic sum of moments of a system of coplanar forces about a moment centre is equal to the moment of their resultant force about the same moment centre."***

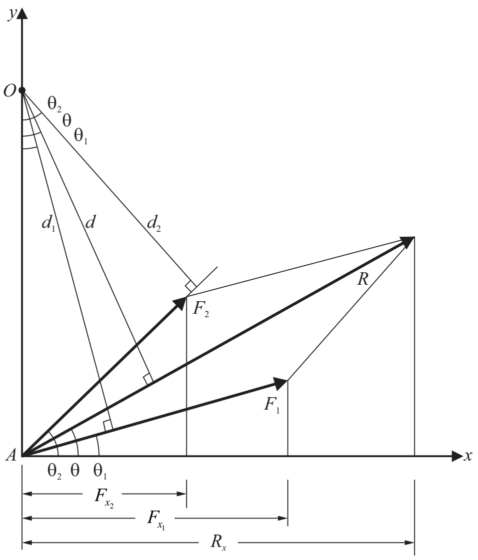


Fig. : Proof of Varignon's theorem

Proof : Consider two forces F_1 and F_2 whose resultant is R as shown in figure. Consider a point O. Perpendicular distance of forces F_1 and F_2 from O are d_1 and d_2 respectively. θ_1, θ_2 and θ are the direction of F_1, F_2 and R from x-axis.

Moment of the force R about O,

$$R \cdot d = R(OA\cos\theta) = OA(R\cos\theta)$$

$$R \cdot d = OA \cdot R_x \quad \text{...(i)}$$

Moment of the force F_1 about O,

$$F_1 \cdot d_1 = F_1(OA\cos\theta_1) = OA(F_1\cos\theta_1)$$

$$F_1 \cdot d_1 = OAF_{x_1} \quad \text{...(ii)}$$

Moment of the force F_2 about O,

$$F_2 \cdot d_2 = F_2(OA\cos\theta_2) = OA(F_2\cos\theta_2)$$

$$F_2 \cdot d_2 = OA \cdot F_{x_2} \quad \text{...(iii)}$$

Adding equation (ii) and equation (iii), we get

$$F_1 \cdot d_1 + F_2 \cdot d_2 = OA \cdot (F_{x_1} + F_{x_2}) \quad \text{...(iv)}$$

But, $R_x = F_{x_1} + F_{x_2}$

The sum of the x-components of the forces F_1 and F_2 is equal to x-components of the resultant R.

Therefore, $OA \cdot (R_x) = OA \cdot (F_{x_1} + F_{x_2})$

So, from equation (i) and equation (iv), we get

$$R \cdot d = F_1 d_1 + F_2 d_2$$

Hence Proved.

Q.3	**(b)**	Discuss various types of supports and beams with sketches.

Ans. **(i)** **Concentrated load :** When a load is applied at a very small area, it is called concentrated load or point load.

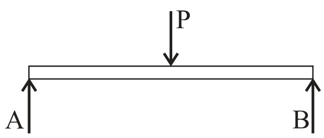


Fig.(a) : Concentrated load

(ii) Distributed load : When a load is spread along the span of beam it is called distributed load. *The intensity (load per unit length) of distributed load is equal to area of load diagram and it acts at the centroid of the load diagram.*

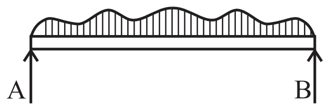


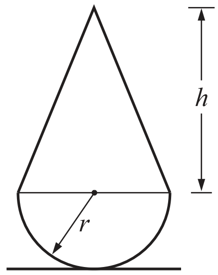
Fig.(b) : Distributed load

Q.4 (a) What are the assumptions made, while finding out the forces in the various members of a framed structure? Discuss the method of section for the analysis of pin-jointed frame.

Ans.
1. The joints of a simple truss are assumed to be connected and frictionless. The joints therefore cannot resist moments.
2. The loads on the truss are applied at the joints only.
3. The members of a truss are straight two fore members with the forces acting collinear with the center line of the members.
4. The weight of the members are negligibly small unless otherwise mentioned.
5. The concept is statically determinate.

Q.4 (b) A body consisting of cone and hemisphere of radius R fixed on the same base rests on a table, the hemisphere being in contact with the table. Find the greatest height of the cone, so that the combined body may stand upright.

Ans.



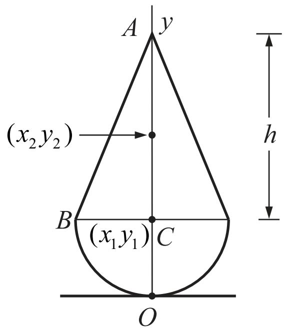
As the body is symmetrical about the vertical axis, its centre of gravity will lie on this axis. Considering two parts of the body, viz. hemisphere and cone. Let bottom of the hemisphere be the axis reference.

Hemisphere :

y-coordinate of center of graving of hemisphere.

$$y_1 = \frac{5}{8} r.$$

Mass of hemisphere :



$$m_1 = \rho \frac{2}{3} \pi r^3$$

For cone :

y-coordinate of centre of gravity of cone

$$y_2 = r + \frac{h}{4}$$

Mass of cone $\quad m_2 = \rho \frac{1}{3} \pi r^2 h$

Distance of centre of gravity of the combined body from O is,

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{\rho\left(\frac{2}{3}\right)\pi r^3 \times \left(\frac{5}{8}\right) r + \rho\left(\frac{1}{3}\right)\pi r^2 h \times \left[r + \left(\frac{h}{4}\right)\right]}{\rho\left(\frac{2}{3}\right)\pi r^3 + \rho\left(\frac{1}{3}\right)\pi r^2 h}$$

$$\bar{y} = \frac{\left(\frac{5}{12}\right) r^4 + \left(\frac{1}{3}\right) r^3 h + \left(\frac{1}{12}\right) r^2 h^2}{\left(\frac{2}{3}\right) r^3 + \left(\frac{1}{3}\right) r^2 h}$$

Condition for stable equilibrium :

Center of gravity of a body should be below the common face BC or maximum it coined with it. Therefore,

$$\bar{y} = \frac{\left(\frac{5}{12}\right) r^4 + \left(\frac{1}{3}\right) r^3 h + \left(\frac{1}{12}\right) r^2 h^2}{\left(\frac{2}{3}\right) r^3 + \left(\frac{1}{3}\right) r^2 h}$$

On solving,

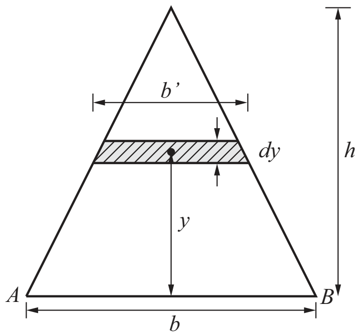
$$3r^2 = h^2$$

$$h = \sqrt{3} \cdot r$$

$$h = 1.739r$$ **Ans.**

Q.5 (a) Derive an expression for movement of inertia of a triangular section about its centroidal axis parallel to base.

Ans. Moment of inertia of a triangle with base width b and height h is to be determined about the base AB.



Consider an elemental strip at distance y from the base AB. Let dy be the thickness of the strip and dA its area. Width of this strip is given by :

$$b' = \frac{(h-y)}{h} \times b$$ [By similarity of triangle] ...(i)

Moment of inertia of this strip about AB

$$I_{AB} = \int_0^h y^2 dA = \int_0^h y^2 b' dy$$

From equation (i), we get

$$I_{AB} = \int_0^h y^2 \frac{(h-y)}{h} \times b \times dy$$

$$I_{AB} = \int_o^h y^2 \frac{(h-y)bdy}{h} = \int_o^h \left(y^2 - \frac{y^3}{h}\right) bdy$$

$$I_{AB} = b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_o^h$$

$$I_{AB} = b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$I_{AB} = \frac{bh^3}{12}$$ **Ans.**

It is clear that the centroidal axis will be parallel to base hence from parallel axis theorem,

$$I_{AB} = I_{xx} + Ay^2$$

Here, y is the distance between base AB and centroidal axis x-x and is equal to $\frac{h}{3}$.

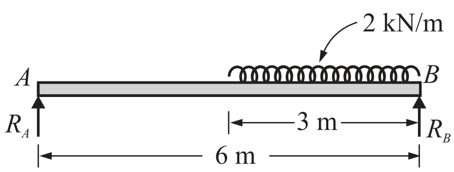
$$\frac{bh^3}{12} = I_{xx} + \frac{1}{2}bh\left(\frac{h}{3}\right)^2 = I_{xx} + \frac{bh^3}{18}$$

Moment of inertia about centroidal axis

$$I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$ **Ans.**

Q.5 (b) A simply supported beam of span 6 m is carrying a uniformly distributed load of 2 kN/m over a length of 3 m from the right end B. Calculate the support reactions.

Ans. FBD :



Calculation for reaction :

$$\Sigma F_y = 0,$$

$$R_A + R_B = 6 \qquad \ldots(i)$$

Talking moment about, A

$\Sigma M_A = 0,$

$$R_B \times 5 + 8 \times 2 \times \left(3 + \frac{3}{2}\right) = 0$$

$$R_B = 4.5\,\text{kN}$$ **Ans.**

From equation (i) ,we get

$$R_A = 6 - 4.5$$

$$R_A = 1.5\,\text{kN}$$ **Ans.**

Q.6 (a) Discuss various basic terms used in dynamics in detail. State general principles in dynamics.

Ans. 1. **Displacement :** It is the shortest distance from the initial to the final position of a point. It is a vector quantity.

Displacement = Final position – Initial position

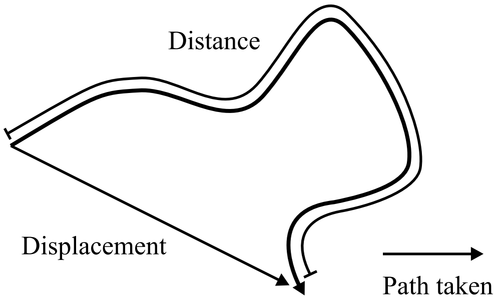


Fig. : Displacement

2. **Distance travelled :** It is the length of path travelled by a particle or body. It is a scalar quantity.
3. **Velocity :** Velocity is the rate of displacement of a body with respect to time.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

4. **Acceleration :** Acceleration is the rate of change of velocity of a body with respect to time.

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time interval}}$$

5. **Average velocity :** It is the average value of the given velocities. Average velocity is displacement over total time.

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

6. **Instantaneous velocity :** It is the velocity at a particular instant of time. It can be obtained from the average velocity by choosing the time interval Δt and the displacement Δx. Instantaneous velocity, (or velocity)

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The unit of velocity is m/s.

7. **Average acceleration :** Let v be the velocity of the particle at any time t. If the velocity becomes $(v + \Delta v)$ at a later time $(t + \Delta t)$ then,

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t}$$

8. **Instantaneous acceleration :** It is the acceleration of a particle at a particular instant of time and can be calculated by choosing the time interval Δt and the velocity Δv.

$$\text{Acceleration } a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}.$$

Acceleration is position if the velocity is increasing. The unit of acceleration is m/s^2.

$$a = \frac{dv}{dt} \quad \text{as} \quad v = \frac{dx}{dt}$$

So, $$a = \frac{d^2 x}{dt^2}$$

Also, $$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} \quad \text{and as } \frac{dx}{dt} = v$$

So, $$a = v\frac{dv}{dx}$$

9. **Uniform motion :** A particle is said to have a uniform motion when its acceleration is zero and its velocity is constant with respect to time. It also called **uniform velocity**.

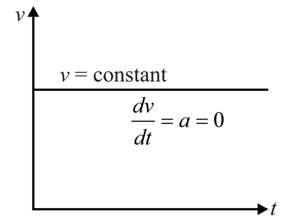


Fig. : Uniform Motion

10. **Uniformly accelerated motion :** A particle moving with a constant acceleration (a = constant with respect to time) is said to be in uniformly accelerated motion.

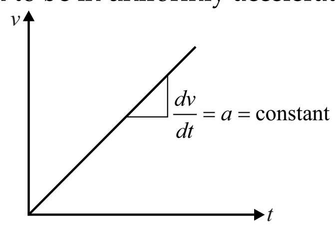


Fig. : Uniformly Accelerated Motion

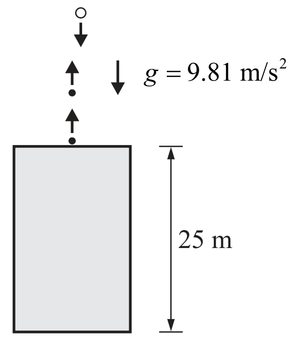
Q.6	**(b)**	A small steel ball is shot vertically upwards from the top of a building 25 m above the ground with an initial velocity of 18 m/sec. Find the total time during which the body is in motion.

Ans. As the ball is being shot vertically. The height of building is not important for calculation.

Given :

Initial velocity $(u) = 18$ m/s

Acceleration due to gravity $(g) = 9.81$ m/s



When ball is shot up :

Using Newton's 3rd equation

$$v^2 = u^2 + 2as \qquad [\because a = -g = -9.81 \text{ m/s}^2]$$

After reaching at peak point, final velocity is zero, so $v = 0$

$$0 = 18^2 - 2 \times 9.81 \times S$$

$$\frac{-18^2}{-2 \times 9.81} = S$$

$$S = 16.51 \text{ m}$$

Time of travel in upward motion

$$S = ut_1 + \frac{1}{2}at_1^2 \qquad [a = g = 9.81 m/s^2,\ u = 18\ m/s]$$

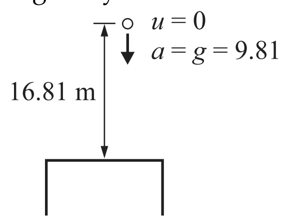
$$16.51 = 18 \cdot t_1 + \frac{1}{2} 9.81 \times t_1^2$$

$$t_1 = 0.76 \text{ s}$$

When the ball is coming down :

It will travel a distance $S = 16.81$ with initial velocity $u = 0$ under the action of gravity.

So, using Newton's second equation,



$$S = ut_2 + \frac{1}{2}at_2^2$$

$$16.81 = 0 \times t_2 + \frac{1}{2} \times 9.81 \times t_2^2$$

$$t_2 = 1.851 \text{ s}$$

Total time taken by ball

$$t = t_1 + t_2 = 0.76 + 1.851$$

$$t = 2.61 \text{ s}$$

Ans.

Q.7 Write short notes on any four of the following :

(a) Fundamental laws of mechanics

Ans. 1. **Newton's law :** Newton's laws consist of "the law of inertia", "the law of motion" and "the law of action and reaction."

Newton's first law (the law of inertia) : The law states that, *"A body will remain at rest or in uniform motion in a straight line unless it is compelled to change this state by forces impressed upon it."*

The first law depicts that if there is no external effect, an object must be still or moving at a constant velocity, which leads to a concept of the net force.

Newton's second law (the law of motion) : This law states that, *"A body acted upon by an external unbalanced force will accelerate in proportion to the magnitude of this force in the direction of applied force"*.

i.e. $F \propto a$

The second law quantifies the force in terms of acceleration and mass of the object.

$$F = m\,a$$

Newton's third law (the law of action and reaction) : This law states that, *"For every action (or force) there is an equal and opposite reaction (or force)"*.

The third law illustrates the existence of the counter force which is related to normal forces and tension, etc.

2. **Law of conservation of mass :** This law states that, *"Mass can neither be created nor be destroyed through any physical or chemical process"*. Mathematical expression of this law is :

$$\frac{d}{dt}(m) = 0$$

Where, m is mass of a body.

3. **Law of conservation of energy :** This law states that, *"Energy can neither be created nor be destroyed, it can only transform from one form to another"*.

Q.7 (b) Bow's Notation

Ans. 1. When the different system of the forces are drawn, then spaces are formed around it. These spaces so formed are named by capital letters *A, B, C, D, E* and so on in order.

2. This method of putting the capital, alphabetic letters on either side of the forces in order is called as Bow's notation.

3. Bow's notation is used to represent or designate or to denote the force in graphical solution of the problem.

For example, consider a force of 30 N is acting on a body. Two spaces are formed around it are named by Bow's notation *A, B* as shown in figure.

To represent this force in a force diagram or vector diagram, a suitable scale is taken (1 cm = 10 N as in figure) and line '*ab*' = 3 cm is drawn parallel to the line of action of *AB* as shown in figure. Length '*ab*' shows the magnitude of a force and an arrow head indicate direction.

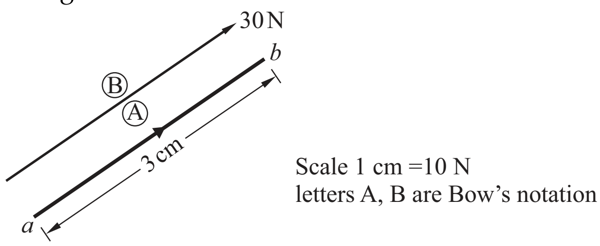


Fig. : Representation of a force by Bow's notation

Q.7 (c) Types of loading on beam

Ans. **(i) Concentrated load :** When a load is applied at a very small area, it is called concentrated load or point load.

(ii) Distributed load : When a load is spread along the span of beam it is called distributed load. *The intensity (load per unit length) of distributed load is equal to area of load diagram and it acts at the centroid of the load diagram.*

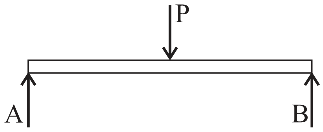


Fig.(a) : Concentrated load

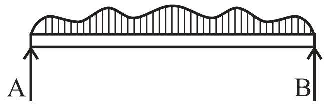


Fig.(b) : Distributed load

Q.7 (d) Parallel axis theorem

Ans. Moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axes. Referring to figure, the above theorem means :

$$I_{AB} = \bar{I} + Ay^2$$

Where, I_{AB} = Moment of inertia about axis AB.

$\bar{I}$ = Moment of inertia about centroidal axis GG parallel to AB.

A = The area of the plane figure given.

y = The distance between the axis AB and the parallel centroidal axis GG.

Proof : Consider an elemental parallel strip dA at a distance y from the centroidal axis as shown in figure,

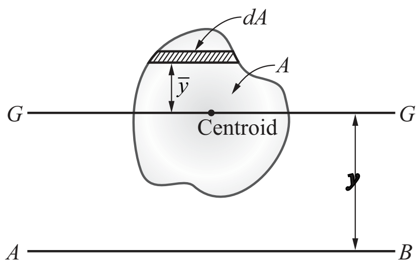


Fig.

Then, $$I_{AB} = \Sigma(\bar{y} + y)^2 dA$$

$$I_{AB} = \Sigma(\bar{y}^2 + 2\bar{y}y + y^2) dA$$

$$I_{AB} = \Sigma\bar{y}^2 dA + \Sigma 2\bar{y}y\, dA + \Sigma y^2\, dA$$

Now, $\Sigma\bar{y}^2 dA$ = Moment of inertia about the centroidal axis $= \bar{I}$

$\Sigma 2\bar{y}y\, dA$ can be written as,

$$\Sigma 2\bar{y}y\, dA = 2y\Sigma\bar{y}\, dA = 2yA\frac{\Sigma\bar{y}\, dA}{A}$$

In the above term $2yA$ is constant and $\frac{\Sigma\bar{y}dA}{A}$ is the distance of centroid from the reference axis GG.

Since GG is passing through the centroid itself $\frac{\Sigma\bar{y}dA}{A}$ is zero and hence the term $\Sigma 2y\bar{y}\, dA$ is zero.

Now, the third term,

$$\Sigma y^2\, dA = y^2 \Sigma dA = Ay^2$$

Therefore, $$I_{AB} = \bar{I} + Ay^2$$

Q.7 (e) Types of motion

Ans. **Types of motion :**

A body may move in any direction in space. In this chapter, only motion in a single plane is considered. This type of motion is called plane motion. Plane motion may be classified as,

(i) Translation : A motion is said to be translation, if a straight line drawn on the moving body remains parallel to its original position at any time. During translation if the path traced by a point is a straight line, it is called rectilinear translation (Fig.1(a)) and if the path is a curve one it is called curvilinear translation (Fig.1(b)).

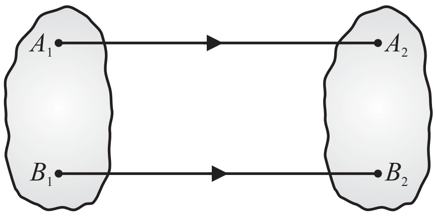


Fig.1(a) : Rectilinear translation

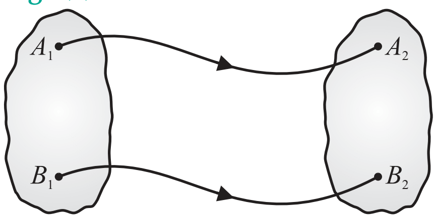


Fig.1(b) : Curvilinear translation

In the study of the motion of particles, rectilinear translation and curvilinear translation are usually referred as linear motion and curvilinear motion.

(ii) Rotation : A motion is said to be rotation if all particles of a rigid body move in a concentric circle.

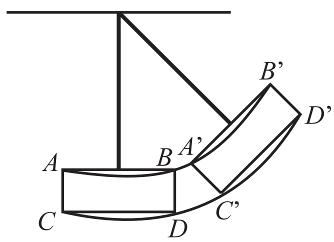


Fig.2 : Rotation

General plane motion : There are many other types of plane motion, i.e., motion in which all the particles of the body move in parallel planes. Any plane motion which is neither a rotation nor a translation is referred to as a general plane motion. Two examples of general plane motion are shown in Fig.5.

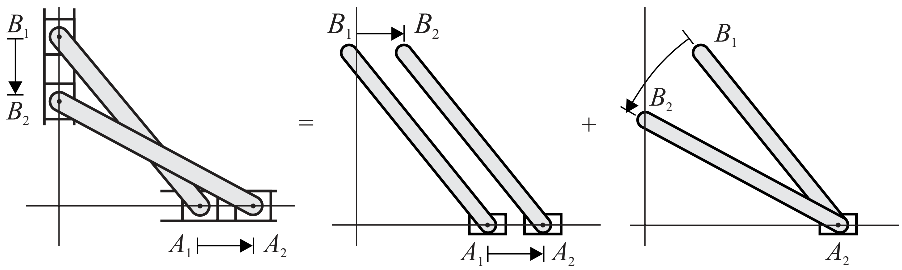
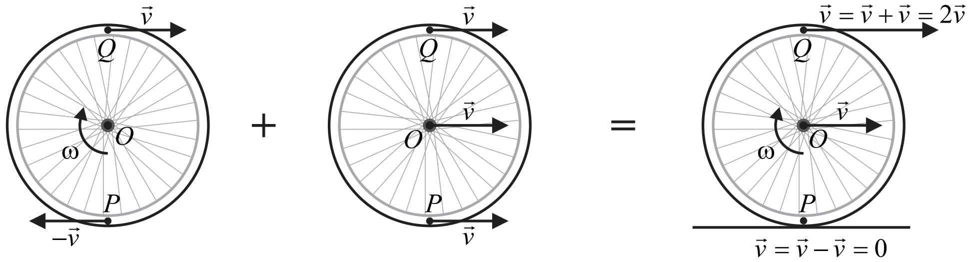


Fig.5 : General plane motion

(iv) Rolling : Rolling motion is a combination of both translation and rotation. Common examples of such motion are points on wheels of moving vehicles, a ladder sliding down from its position against wall etc. It is also a kind of general plane motion.



(a) Pure rotation (b) Pure translation (c) Rolling motion

Fig.6 : Rolling motion

In pure rolling motion the velocity at point of contact between wheel and surface is zero. It is also called rolling without slip.

(v) Space motion : It is the motion of body in 3D space .
Example : Motion of satellite.

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