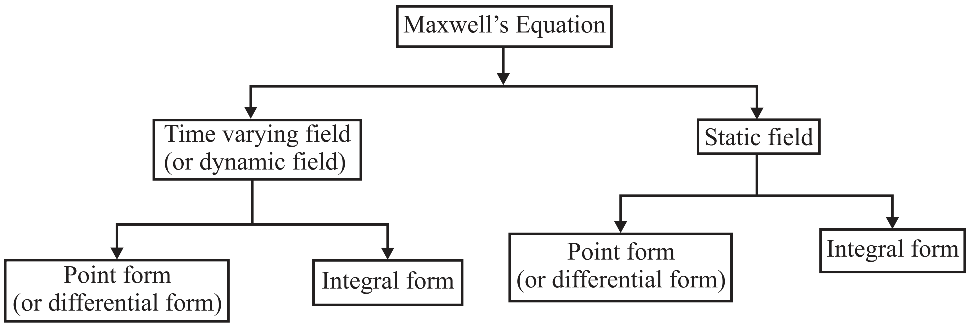
January : 2016 (CBCS)

Note : **Max. marks : 60**

Attempt any five questions. All questions carry equal marks.

Q.1 **(a)** Deduce Maxwell's equation for free space and prove that the electromagnetic waves are transverse.

Ans. **Maxwell's equation for free space :**



Maxwell's equations for dynamic fields :

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_s \vec{D} \cdot d\vec{S} = \int_v \rho_v dv$	Gauss's law for electric field
$\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot d\vec{S} = 0$	Gauss's law for magnetic field
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} d\vec{S}$	Faraday's law of electromagnetic induction
$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J}_c + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$	Ampere's law

Maxwell's Equations for Static Fields :

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_s \vec{D} \cdot d\vec{S} = \int_v \rho_v dv$	Gauss's law
$\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot d\vec{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Conservative nature of electrostatic field and Kirchoff's law
$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{S}$	Ampere's law

Proof for electromagnetic waves :

Transverse nature of electromagnetic waves can be proved with the help of Maxwell's equation $\text{div}\,\vec{E} = 0$ and $\text{div}\,\vec{H} = 0$ for free space.

Let $\vec{E} = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

$$\nabla \cdot \vec{E} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left[(\vec{E}_{0x}\hat{i} + \vec{E}_{0y}\hat{j} + \vec{E}_{0z}\hat{k}) e^{i(k_x x + k_y y + k_z z - \omega t)} \right]$$

$$= \frac{\partial}{\partial x}\left[\vec{E}_{0x} e^{i(k_x x + k_y y + k_z z - \omega t)}\right] + \frac{\partial}{\partial y}\left[\vec{E}_{0y} e^{i(k_x x + k_y y + k_z z - \omega t)}\right] + \frac{\partial}{\partial z}\left[\vec{E}_{0z} e^{i(k_x x + k_y y + k_z z - \omega t)}\right]$$

$$= \left[\vec{E}_{0x} ik_x + \vec{E}_{0y} ik_y + \vec{E}_{0z} ik_z\right] e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= i\left[\vec{E}_{0x} k_x + \vec{E}_{0y} k_y + \vec{E}_{0z} k_z\right] e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i\left(\vec{k}\,\vec{E}_0\right) e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i\vec{k}\cdot\vec{E}$$

$\therefore$ For free space $\nabla\cdot\vec{E} = 0$

$$i\vec{k}\cdot\vec{E} = 0 \text{ or } \vec{k}\cdot\vec{E} = 0$$

It means the wave vector $\vec{k}$ is perpendicular to $\vec{E}$. Similarly, from $\nabla\cdot\vec{H} = 0$ it can be shown that $\vec{k}\cdot\vec{H} = 0$.

Hence, the wave vector $\vec{k}$ is perpendicular to $\vec{H}$. Therefore, the relations $\vec{k}\cdot\vec{E} = 0$ and $\vec{k}\cdot\vec{H} = 0$ indicate that the electromagnetic field vectors $\vec{E}$ and $\vec{H}$ (or $\vec{B}$, as $\vec{B} = \mu_0\vec{H}$) both are perpendicular to the direction of propagation vector $\vec{k}$. It means that the electromagnetic waves are transverse in nature.

Q.2 (b) State and prove Stoke's theorem.

Ans. Stoke's theorem :

The Stoke's theorem states that, "The line integral of a vector $\vec{A}$ around a closed path L is equal to the integral of curl of $\vec{A}$ over the open surface S enclosed by the closed path L".

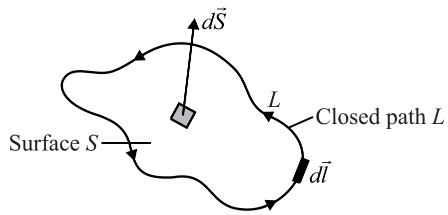


Fig.1

This theorem is applicable only when $\vec{A}$ and $\nabla\times\vec{A}$ are continuous on the surface S.
By applying this theorem, line integral is converted into surface integral.
Mathematically Stoke's theorem is given by,

$$\int_S (\nabla\times\vec{A})d\vec{S} = \oint_{\text{boundary}} \vec{A}\cdot dl$$

Proof :

Consider an open surface S which is broken into 'n' incremental surface area, $\Delta S_1, \Delta S_2 .... \Delta S_n$ or area ΔS.

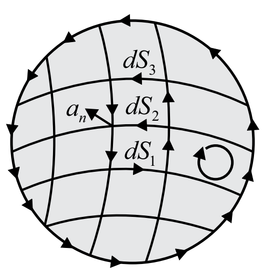


Fig.2

Applying the definition of curl of a vector field the incremental surface area ΔS_1.

$$\nabla\times\vec{A} = \lim_{\Delta S_1\to 0}\oint\frac{\vec{A}\cdot d\vec{l}_{\Delta S_1}}{\Delta S_1}\vec{a}_n \text{ where } \vec{a}_n = \text{unit vector normal to } \Delta S_1.$$

Now,
$$\nabla\times\vec{A} = \lim_{\Delta S_1\to 0}\frac{\oint\vec{A}\cdot d\vec{l}_{\Delta S_1}}{\Delta S_1}$$

$$(\nabla\times\vec{A})\Delta S_1 = \oint\vec{A}\cdot d\vec{l}_{\Delta S_1}$$

Similarly,
$$(\nabla\times\vec{A})\Delta S_2 = \oint\vec{A}\cdot d\vec{l}_{\Delta S_2}$$
$$\vdots \qquad\qquad \vdots$$
$$(\nabla\times\vec{A})\Delta S_n = \oint\vec{A}\cdot d\vec{l}_{\Delta S_n}$$

Adding all these equation, we get

$$(\nabla\times\vec{A})\Delta S_1 + (\nabla\times\vec{A})\Delta S_2 + (\nabla\times\vec{A})\Delta S_3 + \ldots\ldots\ldots(\nabla\times\vec{A})\Delta S_n$$
$$= \oint\vec{A}\cdot d\vec{l}_{\Delta S_1} + \oint\vec{A}\cdot d\vec{l}_{\Delta S_2} + \oint\vec{A}\cdot d\vec{l}_{\Delta S_3} + \ldots\ldots\oint\vec{A}\cdot d\vec{l}_{\Delta S_n} \quad \ldots\text{(i)}$$

As we find the closed line integral for each incremental area, some cancellation will occur because, boundary is getting traced in two opposite directions. Hence summation of all closed line integral for each and every ΔS ends up in a single closed one integral to be obtained for the outer boundary of the total surface S. Hence the equation (i) becomes,

$$\int_S(\nabla\times\vec{A})\,d\vec{S} = \oint_{\text{boundary}}\vec{A}\cdot d\vec{l}$$

Hence Proved.

Q.2 **(a)** What do you mean by laser and its working principle, important requirements and applications?

Ans. **Laser :**

The word LASER is an acronym for **"Light Amplification by Stimulated Emission of Radiation"**. It is a device to produce an intense monochromatic, collimated and highly coherent beam of light and depends on the phenomenon of stimulated emission.

Working principle :

1. **Absorption :** Suppose an atom is in the lower energy level E_1. If a photon of energy $h\nu = (E_2 - E_1)$ is incident on the atom, it imparts its energy to the atom and disappears. Then we say that the atom absorbed the incident photon. As a result of absorption of energy, the atom jumps to the excited sate E_2 (Fig.1). Such a transition is called an absorption transition. In each absorption transition event, an atom in the medium is excited and one photon is subtracted from the incident light beam. We may express the absorption transition as,

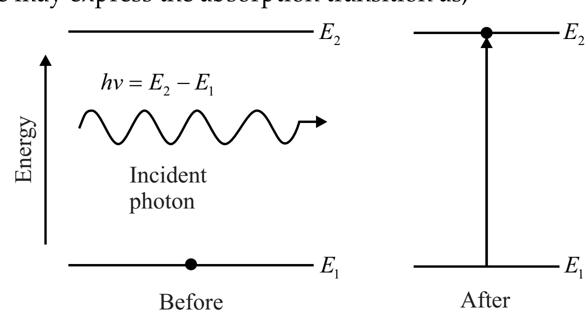


Fig.1 : Absorption process

$$A + h\nu = A^*$$

Where, A is an atom in the lower state and A^* is the corresponding excited atom.

2. **Spontaneous emission :** When an atom at lower energy level is excited to a higher energy level, it cannot stay in the excited state for a relatively longer time. In a time of about 10^{-8} s, the atom

reverts to the lower energy state by releasing a photon of energy $h\nu (h\nu = E_2 - E_1)$. The emission of photon occurs on its own and without any external impetus given to the excited atom (Fig.2). Emission of a photon by an atom without any external impetus is called spontaneous emission. We may write the transition as,

$$A^* \rightarrow A + h\nu$$

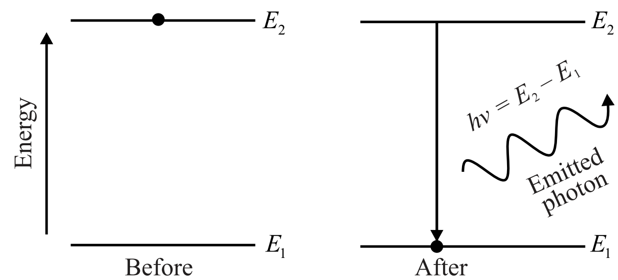


Fig.2 : Spontaneous emission process

3. **Stimulated emission :** If an atom in the excited state interacts with a photon of energy $h\nu = E_2 - E_1$. The photon induces the excited atom to make a downward transition, well before the atom can make a spontaneous transition. The electric field of the light wave interact with the electrons of the excited atom and triggers stimulated emission. As a result, the excited atom emits the excess energy in the form of photon, $h\nu = E_2 - E_1$ and returns to the lower energy state. The passing photon is not affected while the exited atom emits a photon (Fig.3). The phenomenon of forced photon emission by an excited atom due to the action of an external agency is called stimulated emission. It is also known as induced emission. The transition may be written as

$$A^* + h\nu \rightarrow A + 2h\nu$$

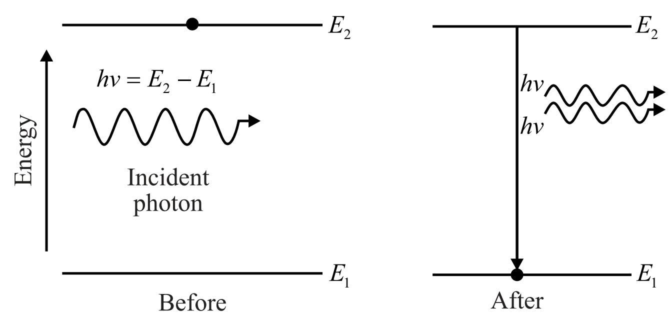


Fig.3 : Stimulated emission process.

Requirement for amplification of light :

Population inversion : The number of atoms per unit volume occupying a certain energy state is called population.

Under ordinary condition of thermal equilibrium the number of atoms in higher energy level (N_2) is considerably smaller than the number of atoms in lower (ground) energy level (N_1). So there is very little stimulated emission compared to absorption. Hence under ordinary condition an incident photon is more likely to be absorbed rather than emission. Hence laser action will not take place.

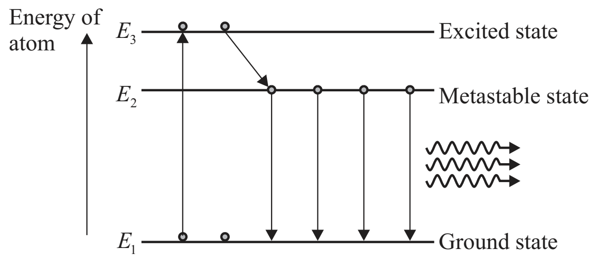


Fig.4 : Population inversion

If however the large number of atoms made available in the higher energy level (i.e. $N_2 > N_1$) then stimulated emission takes place easily.

If the population of higher energy level is more than the population of the lower energy level (i.e. $N_2 > N_1$), then it is said to be **inverted population** or **population inversion.**

Population inversion is a necessary condition for stimulated emission. For a material in thermal equilibrium, the distribution of electrons in various energy state is given by the Boltzmann distribution law :

$$N_2 = N_1 \exp\left[\frac{-(E_2 - E_1)}{kT}\right]$$

Where N_1 and N_2 are the atom densities in state 1 and 2 with energies E_1 and E_2 respectively. T and k are the absolute temperature and Boltzmann constant, respectively.

According to Boltzmann law, higher energy states are the least populated and the population of atoms in the higher energy states decreases exponentially with energy.

The process of achieving the larger number of atoms in the higher energy level than the lower energy level is known as **"Population Inversion"**.

Applications :

Due to the unidirectional and monochromatic properties, a laser beam find its vast uses, some of the application of laser are :

1. **In measurement of very long distance :** The light beam obtained from laser is unidirectional, therefore it is used in measuring very long distances.
2. **In making fine holes and cutting the thick metal sheets :** The beam obtained from laser is very intense and unidirectional, therefore, if the laser beam is focused by a convex lens, its entire energy will concentrate in a very small area ($\simeq 10^{-10}\,\text{m}^2$).

   As a result at that point intensity of beam becomes so strong that the material placed at that point melts and get evaporated and thus a hole is obtained at that point.
3. **In telecommunication :** The laser beam is not absorbed by water. Hence laser beam can be used to transmit a number of television programs simultaneously though water. Since lasers are monochromatic, more messages can be modulated to transmit. Lasers are used in fiber optics communication systems.
4. **In photography :** A laser beam can be used to obtain 3-D images without using the lenses, i.e. storage of three dimensional image on two dimensional photo graphic plate (holography).
5. **In medical science :** Lasers are used in eye surgeries as they can be focused to very fine spot. It is used to weld detached retinas.

   Bloodless cancer surgery can be performed with lasers as the beam can be focused on a small area and only the harmful tissues can be destroyed without damaging the surrounding region. Lasers are used in dental surgeries and plastic surgeries for treating skin diseases.

Q.2 (b) Explain with the help of a neat diagram the principle and working of a He-Ne laser.

Ans. **He-Ne laser :** Helium Neon laser was the first successful gas laser. It is a four level laser.

He-Ne laser is a gas laser in which a mixture of He-Ne gases is used. The reason is that some of excited energy state of He are very close to excited state of Ne gas. He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics. Its usual operation wavelength is $6328\,\mathring{A}$, in the red portion of the visible spectrum.

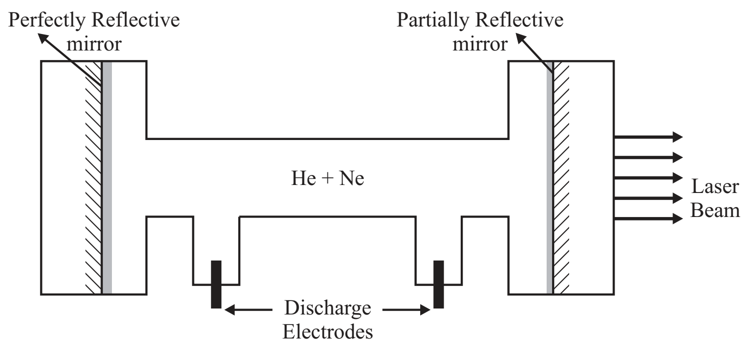


Fig.1 : He-Ne laser

Its main parts are :

1. **Working substances :** The active medium of He-Ne gas laser is a mixture of He and Ne gases in the ratio 10 : 1 at low pressure 1 Torr.
2. **Resonant cavity :** It is a discharge tube of length nearly 50 cm and diameter 1 cm with its ends perfectly plane and parallel. The left end of tube is a perfect reflector while right end is a partial reflector.
3. **Exciting source :** Population inversion is achieved by electric discharge.

Construction : It consists of a long discharge tube filled with He-Ne gases. One end of tube is provided with a perfect reflector and other end with a partial reflector. The pumping of gas mixture is done by electric discharge.

An electric discharge is produced in the gas by electrodes outside the tube connected to a source of high frequency alternating current.

Principle and working :

The energy level diagram of He-Ne laser is shown in Fig.2.

1. When the power is switched ON, the e^- from the discharge collide and pump the He and Ne atoms to metastable states 20.61 eV and 20.66 eV respectively above their ground state.

   Transition : $F_1 \rightarrow F_2$ Absorption.

2. Some of the excited He atom transfer their energy to Ne atom in collision with 0.05 eV of additional energy.

   Since the purpose of the He atom is to excite Ne atom and achieving a population inversion in the Ne-atom.

   Energy transfer from, $\text{He} \rightarrow \text{Ne}$.

3. When an excited Ne atom passes from metastable state at 20.66 eV to an excited state of 18.70 eV, it emits a photon of wavelength $6328\,\mathring{A}$. This photon travels through the gas mixture and if it is moving parallel to the axis of tube, is reflected back and forth by the mirror-ends until it stimulates an excited Ne atom and causes it to emit a $6328\,\mathring{A}$ photon in phase with stimulating photon.

   The stimulated transition from 20.66 eV to 18.70 eV level is the laser transition.

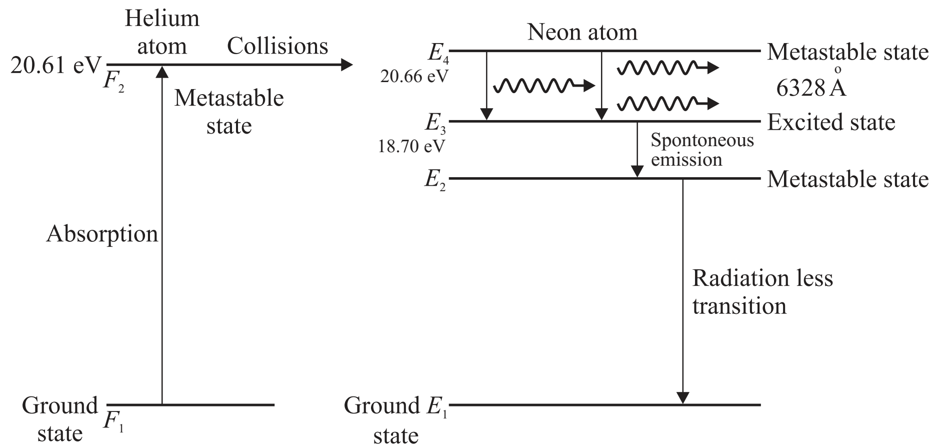


Fig.2 : Energy level diagram of He-Ne laser

The process is continued and a beam of coherent radiation becomes sufficiently intense.

Transition : $E_4 \rightarrow E_3$ Stimulated emission.

$20.66\,\text{eV} \rightarrow 18.70\,\text{eV}$ (Laser transition)

4. The Ne atom passes from the 18.70 eV level, spontaneously to a lower metastable state emitting incoherent light.

Transition : $E_3 \rightarrow E_2$ Spontaneous emission.

5. Finally the Ne atom comes down to the ground state through collision with the tube walls. This transition, from lower metastable state E_2 to the ground state E_1 is radiation-less transition.

Transition : $E_2 \rightarrow E_1$ radiation-less transition.

Since the E_2 of neon is a metastable state then there is possibility of atoms in the level E_2 getting re-excited again to higher level E_3. If this happens then it may disturb the population inversion at the level E_4.

This can be protected by reducing the diameter of the tube (i.e. narrow tube) so that atoms in the energy state E_2 follow the transition directly to the lower energy level E_1 mainly through the collision with the walls of tube instead of exciting atoms from the level E_2 to the level E_3.

Q.3 (a) Describe schematically the basic element of optical fibres communication system.

Ans. **Basic element of optical fibre communication system :**

Basic block diagram of optical fibre communication system consists of following important blocks.

(i) Transmitter (ii) Information channel (iii) Receiver.

Figure shows block diagram of OFC system.

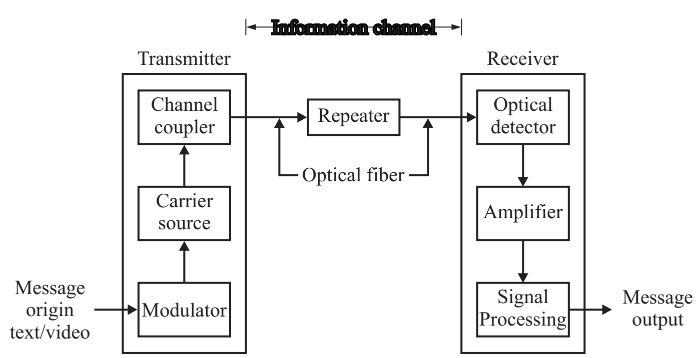


Fig. : Block diagram of OFC systems

1. Message origin :

Generally message origin is from a transducer that converts a non-electrical message into an electrical signal. Common examples include microphones for converting sound waves into current and video (TV) cameras for converting image into current. For data transfer between computers, the message is already in electrical form.

2. **Modulator :**

The modulator has two main functions :

(i) It converts the electrical message into the proper format.

(ii) It impresses this signal onto the wave generated by the carrier source.

Two distinct categories of modulation are used i.e. analog modulation and digital modulation.

3. **Carrier source :**

Carrier source generates the wave on which the information is transmitted. This wave is called the carrier. For fibre optic system, a laser diode (LD) or a light emitting diode (LED) is used. They can be called as optic oscillators, they provide stable, single frequency waves with sufficient power for long distance propagation.

4. **Channel coupler :**

Coupler feeds the power into the information channel. For an atmospheric optic system, the channel coupler is a lens used for collimating the light emitted by the source and directing this light towards the receiver. The coupler must efficiently transfer the modulated light beam from the source to the optic fibre. The channel coupler design is an important part of fibre system because of possibility of high losses.

5. **Information channel :**

The information channel is the path between the transmitter and receiver. In fibre optic communications, a glass or plastic fibre is the channel. Repeaters can be used only for digital systems. They convert weak and distorted optical signals to electrical ones and then regenerate the original digital pulse trains for further transmission.

6. **Optical detector :**

The information being transmitted is detected by detector. In the fibre system the optic wave is converted into an electric current by a photodetector. The current developed by the detector is proportional to the power in the incident optic wave. Detector output current contains the transmitted information. This detector output is then filtered to remove the constant bias and then amplified.

7. **Signal processing :**

Signal processing includes filtering, amplification. Proper filtering maximizes the ratio of signal to unwanted power. For a digital system decision circuit is an additional block. The bit error rate (BER) should be very small for quality communications.

8. **Message output :**

The electrical form of the message emerging from the signal processor are transformed into a sound wave or visual image. Sometimes these signals are directly usable when computers or other machines are connected through a fibre system.

Q.3 (b) What are optical fibres? Give their classification.

Ans. **Optical fibre :** An optical fibre is a thin transparent flexible strand (wire) that consists of a core surrounded by cladding. It confines electromagnetic energy in the form of light within its surfaces and guides the light in a direction parallel to its axis.

Structure : Optical fibre consists of following three sections :

1. The inner most layer or the **core** where the light travels. It made of glass or plastic.
2. The fibre core is surrounded by a transparent sheath called **cladding**. It is a glass or plastic coating having optical properties much different from those of the core. It has a lower index of refraction to keep the light in the core.

3. These layers are protected by an outer section called **sheath**. It is made of polymer or any other material that provides protection against moisture, crushing and other environmental dangers. It has no optical properties affecting the propagation of light within the fibre.

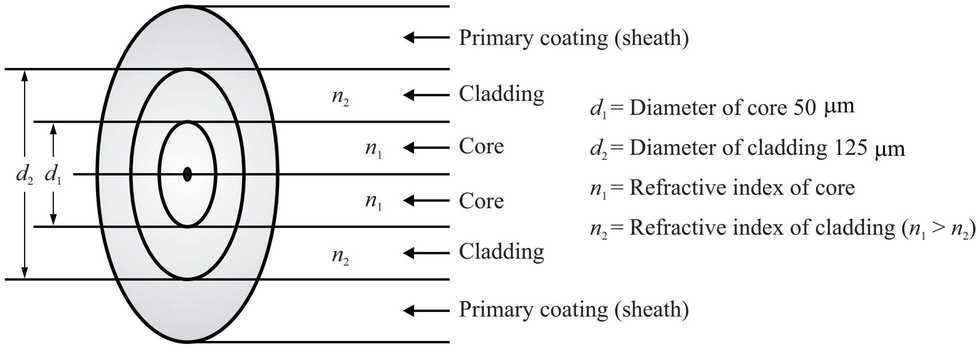


Fig.1 : Internal structure of optical fibre

Classification :

Optical fibres are classified into two categories on the basis of refractive index profile.

1. Step index fibres
2. Graded index (GRIN) fibres

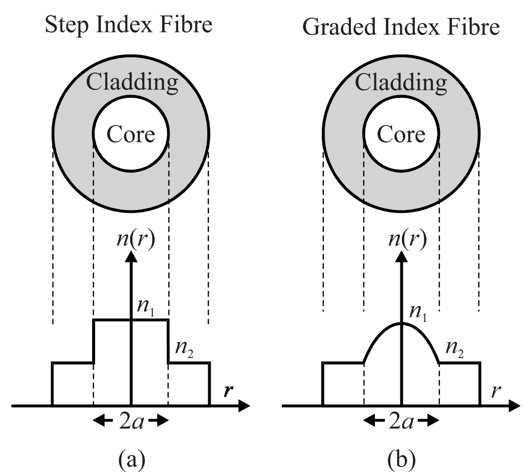


Fig.2 : Classification of optical fibres into (a) step index and (b) graded index based on refractive index profile of the core and cladding.

Step index fibre : In step index fibre the refractive index of the core is constant along the radial direction and abruptly falls to a lower value at the cladding and core boundary (see Fig.2(a)).

Graded index fibre : In case of GRIN fibres, the refractive index of the core is not constant but varies smoothly over the diameter of the core (see Fig.2(b)). It has a maximum value at the center and decreases gradually towards the outer edge of the core. At the core-cladding interface the refractive index of the core matches with the refractive index of the cladding. The refractive index of the cladding is constant.

On the basis of the modes of light propagations, optical fibres are classified into two categories as

(i) The single mode fibre (SMF) (ii) The multimode fibre (MMF)

Single mode fibre (SMF) : A single mode fibre has a smaller core diameter and can support only one mode of propagation.

Multimode fibre (MMF) : A multimode fibre has a larger core diameter and can support a number of modes. Multimode fibres are further distinguished on the basis of index profile.

Thus, there are three classification of optical fibres :

(i) Single mode step index fibre (SMSIF)

(ii) Multimode step index fibre (MMSIF)

(iii) Graded index (multimode) fibre (GRIN)

(i) Single mode step index fibre (SMSIF) : A single mode step index fibre has a very thin core of uniform refractive index of a higher value which is surrounded by a cladding of lower refractive index. The refractive index change abruptly at the core-cladding boundary because of which it is known as step index.

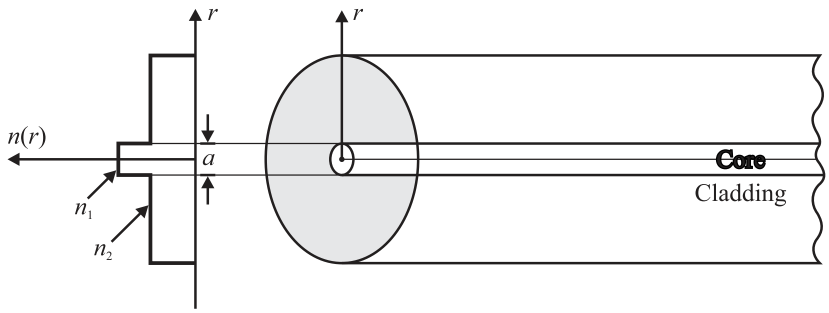


Fig.3 : Single-mode step index fibre

(ii) Multimode step index fibre (MMSIF) : A multimode step index fibre is very much similar to the single mode step index fibre expect that its core is larger diameter. A typical fibre has a core diameter of $100\,\mu m$, which is very larger compared to the wavelength of light being transmitted. Light follows zig-zag path inside the fibre. Many such zig-zag paths of propagation are permitted in a MMF. The NA (Numerical Aperture) of a MMF is larger as the core diameter of the fibre is larger. It is the order of 0.3.

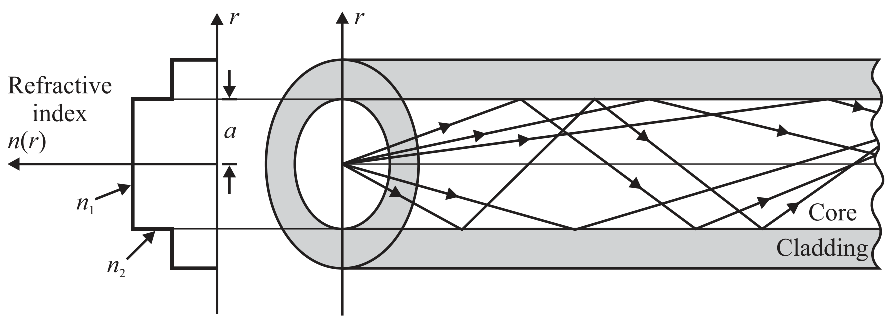


Fig.4 : Multimode step index fibre

(iii) Graded index (multimode) fibre (GRIN) : A graded index fibre is a multimode fibre with a core consisting of concentric layer of different refractive indices. Therefore the refractive index of core varies with distance from the fibre axis. It has a high value at the centre and falls off with increasing radial distance from the axis. A typical graded index fibre has a core diameter ranging from $50\text{-}85\,\mu m$ and a cladding diameter of $125\,\mu m$.

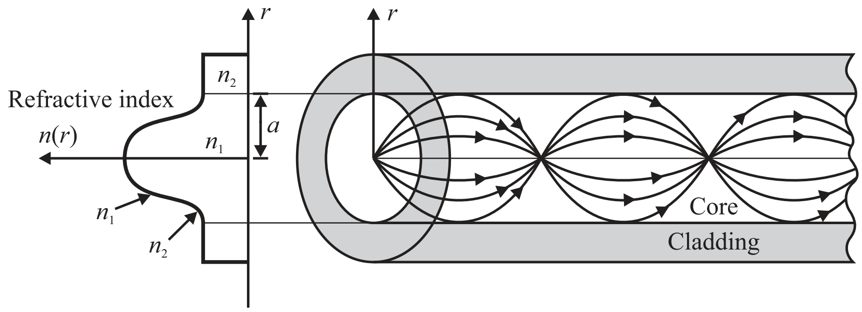


Fig.5 : Graded index fibre

Q.4	(a)	Derive time dependent Schrodinger wave equation.

Ans. **Time dependent Schrodinger wave equation :**

Schrodinger's time dependent wave equation can be obtained from Schrodinger's time independent equation by eliminating energy E.

Consider a particle of mass m moving with velocity v. Let the position of particle at time t be given by (x, y, z).

According to de-Broglie's hypothesis, the particle is associated with a wave system having wavelength λ given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Step I :

Let ψ be the wave variable associated with matter waves. In analogy with the equation

$$\frac{d^2 y}{dt^2} = v^2 \cdot \frac{d^2 y}{dx^2}$$

The differential equation for the matter waves with wave velocity u can be written as,

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right]$$

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \cdot \nabla^2 \psi \qquad \ldots\text{(i)}$$

Where, $\nabla^2 = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$ and is called **Laplacian operator.**

Step II :

The solution of this equation gives us ψ as a function of space and time.

Let $$\psi(x, y, z, t) = \psi_0(x, y, z) \cdot e^{-i\omega t} \qquad \ldots\text{(ii)}$$

Where, $\psi_0(x, y, z) =$ Amplitude of the wave at (x, y, z).

Equation (ii) can be written as,

$$\psi(\vec{r}, t) = \psi_0(\vec{r}) \cdot e^{-i\omega t} \qquad \ldots\text{(iii)}$$

Where, $\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$ and is known as the position vector of point (x, y, z); i, j, k being the unit vectors along the axes.

Step III :

Differentiating equation (iii) w.r.t. t, we have

$$\frac{\partial \psi}{\partial t} = -i\omega\psi_0 \cdot e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i\omega\psi \qquad \ldots\text{(iv)}$$

Step IV :

Now $\omega = 2\pi f$ and $E = hf$

Where, f = Frequency of waves.

$$\omega = 2\pi \cdot \frac{E}{h} = \frac{E}{h/2\pi} \quad \text{or} \quad \omega = \frac{E}{\hbar} \qquad \ldots(v)$$

Where, $\hbar = \dfrac{h}{2\pi}$

Putting this value of ω in equation (iv), we have

$$\frac{\partial \psi}{\partial t} = -i\,\frac{E}{\hbar}\psi \qquad \ldots(vi)$$

Multiplying both sides by i, we have

$$\frac{E}{\hbar}\psi = i\frac{\partial \psi}{\partial t}$$

$$\therefore \quad E\psi = i\hbar\frac{\partial \psi}{\partial t} \qquad \ldots(vii)$$

Step V :

Schrodinger's time independent wave equation is,

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad \ldots(viii)$$

Putting for $E\psi$ from equation (vii) in equation (viii), we have

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial \psi}{\partial t} - V\psi\right] = 0$$

Multiplying both sides by $\dfrac{-\hbar^2}{2m}$, we have

$$\frac{-\hbar^2}{2m}\cdot\nabla^2\psi - i\hbar\frac{\partial \psi}{\partial t} + V\psi = 0$$

$$\therefore \quad \frac{-\hbar^2}{2m}\cdot\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial t}$$

This equation is known as **Schrodinger's time dependent wave equation.**

Q.4 (b) Define phase velocity and group velocity. Show that for non-relativistic free particle the phase velocity is half of the group velocity.

Ans. **Group velocity :** When a number of plane waves of slightly different wavelength travel in the same direction they form wave groups or wave packet. The velocity with which the wave group advances in the medium is known as group velocity.

Phase velocity : The phase velocity of a wave is the rate at which the phase of the wave propagates in space. This is the speed at which the phase of any one frequency component of the wave travels.

For non-relativistic free particle :

A particle whose velocity is less with respect to that of light is called non-relativistic free particle.
According to de-Broglie hypothesis,

$$\lambda = \frac{h}{mv_g}$$

Total energy, $E = \dfrac{1}{2}mv_g^2$

Also, $E = hf$

$$f = \frac{E}{h} = \frac{mv_g^2}{2h}$$

∴ Phase velocity,

$$v_p = f\lambda = \frac{mv_g^2}{2h} \times \frac{h}{mv_g}$$

$$v_p = \frac{v_g}{2}$$

Hence, for non-relativistic free particle the phase velocity is half of the group velocity.

Q.5 **(a)** Explain the mathematical treatment of Newton's rings. Describe an experiment to determine the refractive index of liquid with the help of Newton's rings.

Ans. **Mathematical treatment of Newton's rings :**

Formation of Newton's rings : When a plano convex lens of large focal length is placed in contact with a plane glass plate, a thin air film is formed between the lower surface of the lens and upper surface of the plate. The thickness of the film is very small ($t \approx 0$) near the point of contact and gradually increases from the centre outwards.

If monochromatic light is allowed to fall normally on this film, then alternate bright and dark concentric rings with their centre dark are formed. The interference rings so formed were first investigated by Newton and were called **Newton's Ring**. These rings are used to measure the wavelength of monochromatic light and refractive index of rare liquids.

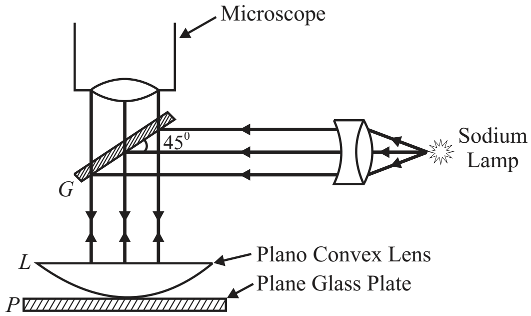


Fig.(a) : Newton's Ring Setup

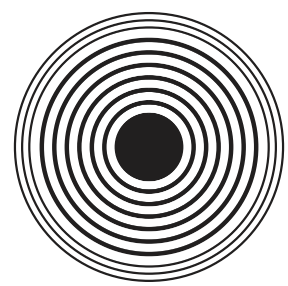


Fig.(b) : Newton's Ring

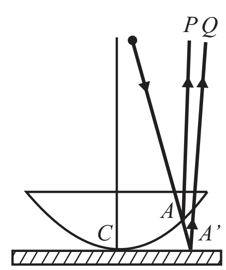


Fig.(c) : Newton's rings by reflected light

Experimental arrangement : The experimental setup is shown in Fig.(a). A plano- convex lens L of large radius of curvature is placed on a plane glass plate P such that both of them is having a point of contact. Light from a monochromatic source is allowed to fall on a glass plate G inclined at an angle of 45^0 to the incident beam. The light reflected from glass plate falls normally on the air film enclosed between L and P.

Reflected light rays superimpose each other and interference takes place. Due to interference of these rays, alternate bright and dark concentric rings are seen with the help of microscope.

The fringes are circular because the air film is symmetrical about the point of contact of the lens with the plane glass plate.

Theory :

As the rings are observed in reflected light, the effective path difference between the interfering rays is given by,

$$2\mu t\cos r+\frac{\lambda}{2} \qquad \text{...(i)}$$

Where, $\mu =$ Refractive index of film (air),

$r =$ Inclination of the ray,

$t =$ Thickness of air film.

For air film, $\mu = 1$

$\because$ Because rays are normally incident.

$\therefore \quad r = 0$

Now, Path difference $= 2t+\frac{\lambda}{2}$ ...(ii)

At $t=0$, Path difference $=\lambda/2$

This is the condition for the minimum intensity. **Hence the centre of Newton's ring is dark.** But sometimes it is observed that centre is bright, this is because a dust particle comes in between the two surfaces at the point of contact.

For maxima (bright ring), the path difference $= n\lambda$

$$\therefore \quad 2t+\frac{\lambda}{2}=n\lambda$$

$$2t=(2n-1)\frac{\lambda}{2} \qquad \text{...(iii)}$$

Where, $n = 1, 2, 3, ...$

For minima (dark ring), the path difference $=(2n+1)\frac{\lambda}{2}$

$$2t+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}$$

$$2t=n\lambda \qquad \text{...(iv)}$$

Where, $n = 0, 1, 2, 3, ...$

As in air-film, t remains constant along a circle with its centre at the point of contact, the fringes are in the form of concentric circles. Since each fringe is a locus of constant thickness of the air-film, these are known as the **"fringe of constant thickness"**.

Determination of refractive index using Newton's ring : The liquid whose refractive index is to be determined is placed between the plano-convex lens and the glass plate of the Newton's ring setup. In case liquid is rarer than glass, a phase change of π will occur at reflection from the lower surface of the liquid film, but if liquid is denser than glass, phase change will occur at reflection from the upper surface of the film. Hence, in both the cases, path difference is equal to $\frac{\lambda}{2}$.

Therefore effective path difference $= 2\mu t\cos r+\frac{\lambda}{2}$

Where, $\mu =$ Refractive index of liquid.

For normal incidence, $r = 0$

$\therefore \quad \cos r = 1$

Effective path difference $= 2\mu t+\frac{\lambda}{2}$

The condition for n^{th} dark ring is given as, $\Delta=(2n+1)\frac{\lambda}{2}$

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t = n\lambda$$

$$2t = \frac{n\lambda}{\mu} \qquad \text{...(i)}$$

From the geometrical property of circle,

$$2t = \frac{r^2}{R} \qquad \text{...(ii)}$$

Where, $r =$ Radius of Newton's ring,

$R =$ Radius of curvature.

From equation (i) and (ii),

$$\frac{r^2}{R} = \frac{n\lambda}{\mu} \quad \Rightarrow \quad r_n^2 = \frac{n\lambda R}{\mu}$$

Where, r_n is the radius of n^{th} dark ring.

If D_n is diameter of n^{th} dark ring,

Then, $$D_n^2 = \frac{4n\lambda R}{\mu} \qquad \text{...(iii)}$$

If D_{n+p} is the diameter of $(n+p)^{th}$ dark ring,

Then, $$D_{n+p}^2 = \frac{4(n+p)\lambda R}{\mu} \qquad \text{...(iv)}$$

Subtracting equation (iii) from equation (iv), we get

$$\left(D_{n+p}^2 - D_n^2\right)_{\text{liquid}} = \frac{4p\lambda R}{\mu} \qquad \text{...(v)}$$

For air film, $\mu = 1$

$$\left(D_{n+p}^2 - D_n^2\right)_{air} = 4p\lambda R \qquad \text{...(vi)}$$

Dividing equation (vi) by equation (v), we get refractive index of liquid,

$$\mu = \frac{\left(D_{n+p}^2 - D_n^2\right)_{\text{air}}}{\left(D_{n+p}^2 - D_n^2\right)_{\text{liquid}}}$$

Q.5 (b) What do you understand by the resolving power of grating?

Ans. **Resolving power of grating :** The ability of a grating to form separate spectral lines of two close wavelength is called its **resolving power**.

Or

The resolving power of a grating is its ability to form separate diffraction maxima of two wavelength that are very close to each other.

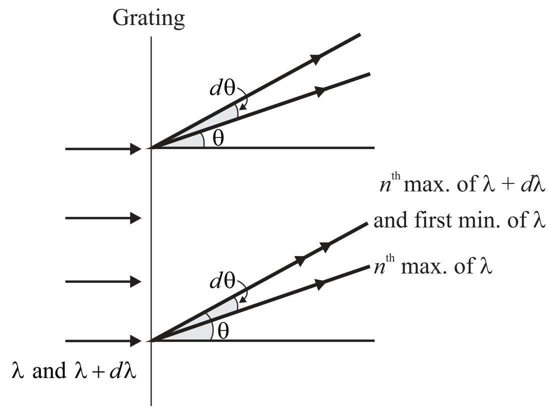


Fig. : Resolving power of grating

If the wavelength of the two spectral lines are λ and $\lambda+d\lambda$, where $d\lambda$ is that minimum difference in wavelengths which is just resolved by the grating at wavelength λ, then the limit of resolution or resolving power of a grating is expressed by the ratio $\frac{\lambda}{d\lambda}$.

i.e. resolving power of grating $=\frac{\lambda}{d\lambda}$ ...(i)

Let a parallel beam of light of wavelength λ and $\lambda+d\lambda$ be incident normally on a diffraction grating.

Let n^{th} principal maxima of wavelength λ be formed in the direction θ. Then we have

$$(e+d)\sin\theta = n\lambda \quad \text{...(ii)}$$

Where, $e=a+b=$ Grating element, $n=$ Order of spectra.

Let the first minima adjacent to the n^{th} maxima be formed in the direction ($\theta+d\theta$). Then from the grating equation of minima,

$$N(e+d)\sin\theta = m\lambda \quad \text{...(iii)}$$

Where, $N=$ Total number of rulings on grating,

m = An integer having values, $m=1, 2, 3,$........, except $0, N, 2N,$.........nN because these value gives zero order, first order, second order, ... principal maxima.

Clearly, the first minima adjacent to the n^{th} principal maxima will be obtained for

$$m = nN+1 \quad \text{...(iv)}$$

Substituting the value of m in equation (iii),

$$N(e+d)\sin(\theta+d\theta) = (nN+1)\lambda$$

$$(e+d)\sin(\theta+d\theta) = \left(\frac{nN+1}{N}\right)\lambda \quad \text{...(v)}$$

According to Rayleigh's criterion, the wavelengths λ and $\lambda+d\lambda$ are just resolved by the grating when the n^{th} maximum of $\lambda+d\lambda$ is also obtained in the direction $(\theta+d\theta)$. Then,

$$(e+d)\sin(\theta+d\theta) = n(\lambda+d\lambda) \quad \text{...(vi)}$$

Comparing equation (v) and equation (vi), we get

$$\frac{nN+1}{N}\lambda = n(\lambda+d\lambda)$$

or $$nN\lambda+\lambda = Nn\lambda+Nnd\lambda$$

or $$\lambda = Nn\,d\lambda$$

or $$\frac{\lambda}{d\lambda} = nN \quad \ldots(vii)$$

As $\frac{\lambda}{d\lambda}$ = resolving power of the diffraction grating.

From equation (vii),

1. The resolving power of grating is equal to the product of the total number of rulings on the grating and the order of the spectrum.
2. The resolving power of grating increases with the order of spectrum.
3. The resolving power increases with the increase in the total number of lines on the grating.
4. The resolving power is independent of the grating element.

Q.6 (a) What is mean by fill factor of a solar cell and what is its significance?

Ans. Fill factor of solar cell :

(i) The open circuit voltage V_{oc} :

It is the output from the solar cell when the load impedance is very high (i.e. $R_L \to \infty$).

(ii) The short circuit current I_{sc} :

It is the current when the load impedance is very small (i.e. $R_L \to 0$) .

(iii) The fill factor (F.F.) :

The product of open circuit V_{OC} and short circuit current I_{SC} gives the ideal power output from the cell.

$$\therefore \quad P_{ideal} = I_{SC} V_{OC}$$

But the actual maximum power output is less than P_{ideal} and is given by the product of maximum power current I_m and maximum power voltage V_m .

$$\therefore \quad P_{\max} = I_m V_m$$

The fill factor of the solar cell is defined as the ratio of the maximum useful power to ideal power.

$$\therefore \quad \text{F.F.} = \frac{I_m V_m}{I_{SC} V_{OC}}$$

The efficiency of solar cell is given by,

$$\eta = \frac{I_m V_m}{\text{Incident light power}} = \frac{I_m V_m}{I_L V_{OC}}$$

Usually the voltage developed by cell is of the order of 0.6 V and the efficiency of conversion of about 15 – 12% for silicon cell.

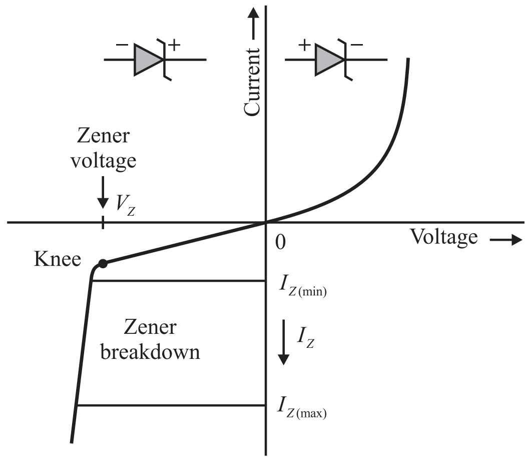
Significance :

1. Higher fill factor solar panel has less parasitic losses, i.e., losses due to the series and parallel resistances within the cell itself.
2. The fill factor can also be used to determine the series resistance of a solar cell.
3. The lower fill factor panel produces less power at its maximum power point compared to the higher fill factor panel.
4. If its fill factor is low (below 0.7), the cells are sold off as Grade-B cells, then sliced, and used for hobbyist use.

Q.6 (b) Explain the *V-I* characteristics of Zener diode.

Ans. The ability to keep the voltage across its terminal constant is the key feature of Zener diode. The Zener diode is called as a **voltage regulated diode** because it maintains a constant output voltage even though the current through it changes.

A Zener diode operating in breakdown region acts as a Zener regulator (Voltage regulator) because it maintains a nearly constant voltage across its terminals.



V-I characteristics of Zener diode :

The V-I characteristic is shown in figure. When a Zener is forward biased its characteristics are just as those of ordinary diode. As the reverse voltage applied to p-n junction is increased, a value is reached at which the current increases greatly from its normal cut off value. This voltage is called a **Zener voltage** V_Z or **breakdown voltage**. So, when a Zener diode is reverse biased, it has sharp breakdown voltage called Zener voltage V_Z.

It is observed from the figure that below knee, the breakdown voltage V_Z remains practically constant. This ability of the diode is called as regulating ability. The regulating ability of Zener diode is an important feature. This feature is used to maintain a constant voltage across its terminal over a specified range of Zener current values [$(I_Z)_{\min}$ and $(I_Z)_{\max}$].

1. There is a minimum value of Zener current $(I_Z)_{\min}$ called breakover current which must be maintained to keep the diode in breakdown or regulation region. If the current is reduced below the knee of the curve, the voltage changes drastically. Now, the regulation is lost.
2. There is a maximum value of Zener current $(I_Z)_{\max}$ above which the Zener may be damaged.

Q.7 **(a)** How does a nucleus behave like a liquid drop? State the postulates of liquid drop model and point out the success and unsuccess of this model.

Ans. **Nuclear liquid drop model :** This model was proposed by Neils Bohr in 1936. He pointed out the strong, short-range, attractive forces between the nucleons that are analogous to those acting between the molecules of liquid. The surface molecules in a liquid drop are attracted by the inner molecules thus generating the surface tension. This surface tension helps the liquid drop to take spherical shape. He proposed almost similar phenomenon to nucleus. The two important properties of a nucleus, the constant density and the constant binding energy per nucleon can be matched with the liquid drop. Thus on the basis of Bohr's liquid drop model the following conclusions may be drawn.

The similarities between the nucleus and a liquid drop are as follows :

1. In the stable state, the nucleus in shape, just like as liquid drop is spherical due to the symmetrical surface tension forces.
2. The force of surface tension acts on the surface of the liquid drop. Similarly there is a potential barrier at the surface of the nucleus.
3. The density of a liquid drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.
4. The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by

bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiation almost immediately.

5. The nuclear forces are short range forces, similarly, as that of liquid, in which the intermolecular forces are short range forces.
6. When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar in which the nucleus break up into two smaller nuclei.

Assumptions :

1. The material of the nucleus is incompressible and the density of all the nuclei is the same.
2. The forces in the nucleus are of :
   (i) Coulomb forces between protons.
   (ii) Powerful attractive nuclear forces.

Merits :

1. It has successfully applied in describing nuclear reactions and explaining nuclear fission.
2. The calculation of atomic masses and binding energies can be done with good accuracy.

Demerits :

1. It cannot explain the existence of magic numbers.
2. It cannot explain why only certain energies of radiations are emitted by a nucleus.
3. It does not explain magnetic moment of nucleus.

Q.7 (b) What are the difference and similarities between nuclear fission and nuclear fusion?

Ans. Similarities : The only similarity between these two types of reactions is that the cause of the production of energy is the same and that is loss in mass (mass defect) i.e. energy is released because of the fact that the sum of masses of the products is less than the sum of the masses of the reactants. The mass which is lost is converted into energy and this energy is generated in the reaction.

Difference between nuclear fission and fusion :

S. No.	Nuclear fission	Nuclear fusion
1.	Nuclear fission is a process in which a heavy nucleus, after capturing a neutron splits up into two lighter nuclei of comparable masses. **Example :** ${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_{0}n^{1} + Q$	Nuclear fusion is a process in which two lighter nuclei combine to produce a heavy and stable nucleus. **Example :** $4{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2{}_{1}\beta^{0} + Q$
2.	The process is possible even at room temperature.	The process is possible only at very high temperature.
3.	The links of the process are neutrons.	The links of the process are protons.
4.	Energy released by the fission of one nucleus of U^{235} is about 200 MeV.	Energy released by the fusion is about 26.7 MeV.
5.	Energy released per nucleon is 0.85 MeV.	Energy released per nucleon is 6.67 MeV.
6.	The fissionable substance is radioactive.	The fusionable substance is not radioactive.

Q.8 Write short note (any two) :
(a) Crystalline and Amorphous solids
(b) Energy band in solid
(c) Hall effect
(d) Compton effect

Ans. (a) Crystalline solids :

If the atoms or molecules in a solid are arranged in some regular fashion then it is known as crystalline solid. Hence, in a crystalline solid, the atoms are arranged in an orderly three-dimensional array that is repeated throughout the structure. The metallic crystals are Cu, Ag, Al etc. The non-metallic crystals are : C, Si, Ge etc.

Amorphous solids :

Amorphous means 'without form'. When the atoms or molecules in a solid are arranged in a regular fashion, then it is known as amorphous solid. The examples are : glass, plastic, rubber etc.

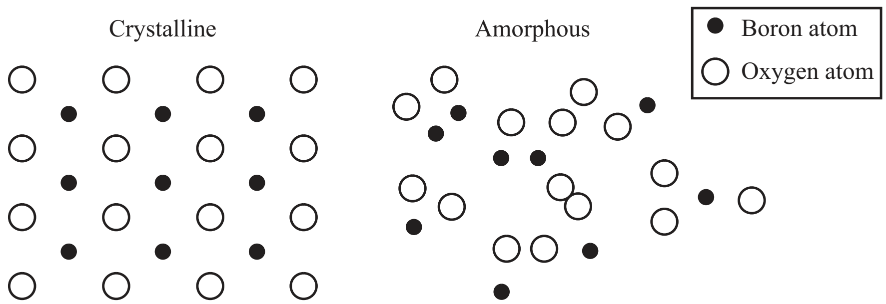


Fig.1 : (a) Crystalline long-range order as well and (b) Amorphous B_2O_3 exhibits only short-range order.

(b) Energy band in solid :

Solids can be classified into conductors, insulators and semiconductors based on their energy band structure.

1. **Conductors :** In conductors, the valence band and the conduction band overlap. There is no forbidden band. The electrons can be made to move and constitute a current by applying a small potential difference. The resistivity of conductors is very low and increases with temperature. Hence, the conductors are said to have a positive temperature coefficient of resistance. The energy band structure is shown in Fig.1.

   Example : Metals like copper, silver, gold, aluminum etc. are good conductors of electricity.

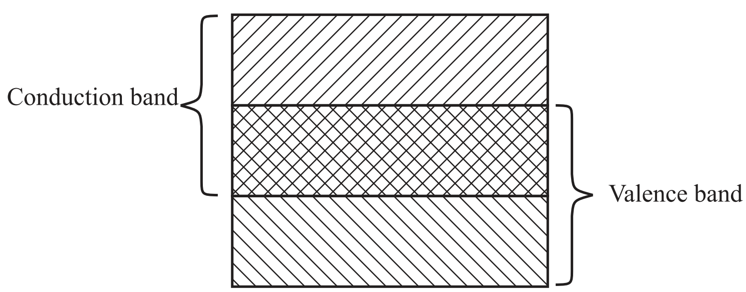


Fig.1 : Energy band of conductors

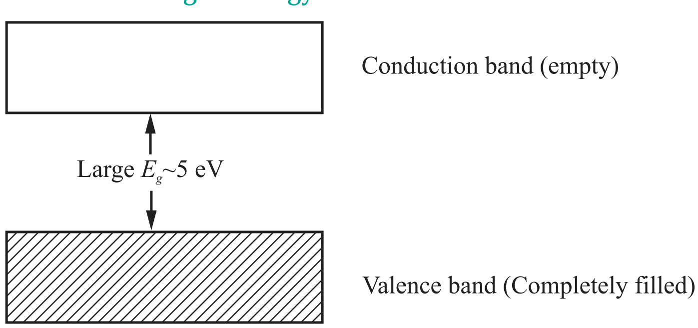


Fig.2 : Energy band of insulators

2. **Insulators :** Insulators have a completely filled valence band and an empty conduction band which are separated by a large forbidden band. The band gap energy is large (of the order of 5 eV). Hence large amount of energy is required to transfer electrons from valence band to conduction band. The insulators have very low conductivity and high resistivity. The energy band structure is shown in Fig.2.

   Example : Diamond, wood, glass etc., are insulators.

3. **Semiconductors :** The term semiconductor applies to a class of material having conductivity intermediate between a conductor and an insulator. In semi-conductors, the valence band is completely filled and the conduction band is empty at absolute zero temperature. The valence band and conduction band are separated by a small forbidden band of the order of 1 eV. Hence, compared to insulators, smaller energy is required to transfer the electrons from valence band to conduction band. Hence, the conductivity is better than insulators but not as good as the conductors. The energy band structure is shown in Fig.3.

   Example : Silicon and germanium are semiconductors having band gap energies of 1.1 eV and 0.7 eV respectively. Some compounds formed between group III and group V elements like gallium arsenide (GaAs) are also semiconductors.

   As temperature is increased, electrons from valence band jump to conduction band leaving vacancy in valence band which is known as hole. The free electrons in conduction band and the holes in valence band take part in conduction. Hence conductivity increases and resistivity decreases with increase in temperature. The semiconductors are said to have a negative temperature coefficient.

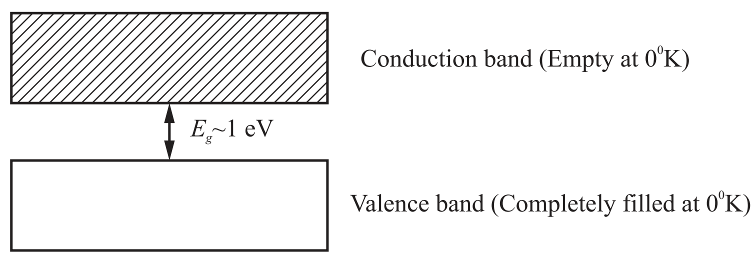


Fig.3 : Energy band of semiconductors

(c) Hall effect : When a metal or a semiconductor carrying a current I is placed in a transverse magnetic field B, a potential difference is produced in the direction normal to both the current and magnetic field directions. This phenomenon is called **Hall effect** and potential difference developed is called **Hall voltage**.

Hall effect measurements showed that it is the negative charge carriers namely electrons which are responsible for electrical conduction in metals. It also showed that there exist two types of charge carriers in semiconductors.

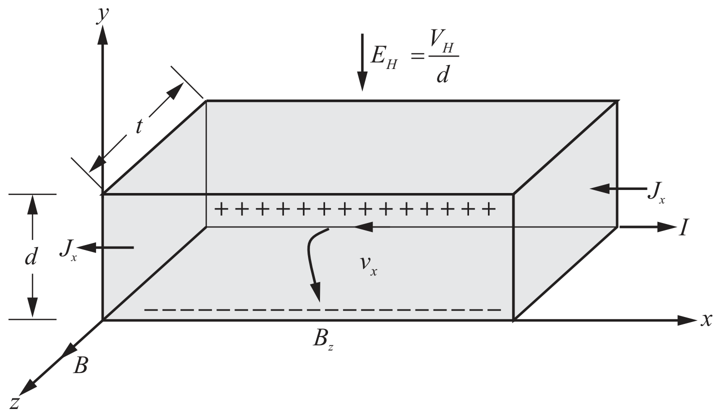
Importance : The importance of Hall effect is that it helps to determine the :

(i) Sign of charge carriers.
(ii) Charge carriers concentration.
(iii) Mobility of charge carriers if conductivity of the material is known.
(iv) Type of semiconductor.

Experimental determination of Hall voltage and Hall coefficient :

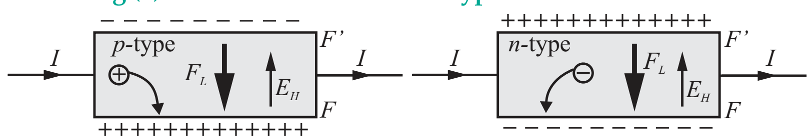
Let us consider an n-type semiconductor in which the conduction is predominated by electrons. Suppose an electric current I flows in the positive x-direction and a magnetic field B is applied normal to this electric field in z-direction as shown in Fig. (a). A force, called the

Lorentz force is exerted on each electron which causes the electron paths to bend. As a result of this, the electrons accumulate on one side of the slab and are deficient on the other side.



(a)

Fig.(a) : Schematic view of an *n*-type semiconductor bar.



(b) **(c)**

Fig. : The direction of magnetic force and Hall field
(b) in *p*-type semiconductor (c) in an *n*-type semiconductor

Thus, an electric field is created in the y- direction which is called the **Hall field**. In equilibrium condition,

Hall force = Lorentz force

$$F_H = F_L$$

$$-qE_H = v_x B_z q \qquad \text{...(i)}$$

Where, v_x is the velocity of the electrons, and q the electronic charge.

$$\therefore \quad E_H = v_x B_z \qquad \text{...(ii)}$$

As current density, $J_x = -Nv_x q$

Putting the value of v_x from equation (ii) in the above equation, we get

$$J_x = -\frac{NE_H q}{B_z}$$

$$N = \frac{-J_x B_z}{qE_H} = -\frac{J_x B_z}{q(V_H / d)} \qquad \text{...(iii)}$$

$$V_H = -\frac{J_x B_z d}{Nq} = -\frac{IB_z d}{NqA} \qquad \text{...(iv)}$$

Where, A is the area of cross-section of end face.

If t is the thickness of the semiconductor specimen, $A = dt$ and the above equation reduces to,

$$V_H = -\frac{B_z I}{Nqt} \qquad \text{...(v)}$$

Hall field per unit current density per unit magnetic induction is called **Hall coefficient** R_H.

Thus,

$$R_H = \frac{E_H}{J_x B_z} = +\frac{V_H / d}{J_x B_z} = -\frac{B_z I}{J_x B_z d N q t}$$

$$R_H = -\frac{1}{Nq} = \frac{1}{\rho} \qquad \text{...(vi)}$$

Where, ρ is the charge density.

In terms of Hall coefficient, Hall voltage is given by,

$$V_H = R_H \frac{BI}{t} \qquad \text{...(vii)}$$

The sign of the Hall coefficient R_H indicates whether electrons or holes predominate in the conduction process.

Experimental determination of mobility :

The electron mobility is given as,

$$\mu_n = \frac{\sigma}{N|q|}$$

Hence, $\mu_n = |R_H| \sigma$

Thus, the magnitude of μ_n can be determined if the conductivity σ has been measured.

Hall angle :

The net electric field E in the semiconductor is a vector sum of E_x (electric field component in x-direction) and E_H (Hall field). It acts at an angle θ_H to the x-axis. θ_H is called the **Hall angle**.

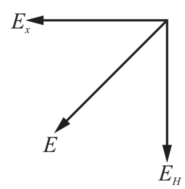


Fig.(d)

From Fig. (d),

$$\tan \theta_H = \frac{E_H}{E_x} \qquad \text{...(viii)}$$

As $$E_H = \frac{V_H}{d} = \frac{B_z J_x}{N|q|} \qquad \text{...(ix)}$$

Also $$E_x = \rho J_x \qquad \text{...(x)}$$

$\therefore$ $$\tan \theta_H = \frac{B_z}{N|q|\rho} \qquad \left[\because \sigma = \frac{1}{\rho}\right]$$

$\therefore$ $$\tan \theta_H = \sigma R_H B_z$$

The product σR_H is designated as μ_n the mobility of electrons.

$\therefore$ $$\tan \theta_H = \mu_n B_z$$

$\therefore$ $$\theta_H = \tan^{-1}(\mu_n B_z) \qquad \text{...(xi)}$$

In the above discussions it is assumed that all carriers travel with a mean speed v_x. However, this does not happen. As a result the value of R_H gets modified. The appropriate value is

$$R_H = \frac{3\pi}{8}\left(-\frac{1}{Nq}\right) \qquad \text{...(xii)}$$

Accordingly,

$$\mu_n = \frac{8}{3\pi}\sigma R_H \qquad \text{...(xiii)}$$

Applications :

1. **Magnetic field meter :** The Hall voltage V_H for a given current is proportional to B. Hence measurement of V_H measures the magnetic field B.
2. **Hall effect multiplier :** This instrument gives an output proportional to the product of two signals. Thus, if current I is made proportional to one input and if B is proportional to the second input, then Hall voltage V_H is proportional to the product of the two inputs.

(d) Compton effect : In 1992 Compton observed that when a monochromatic beam of X-ray was scattered by matter, the scattered beam contained radiation of two different wavelengths, one identical with the incident radiation and the other of longer wavelength.

The change in wavelength depends on the angle of observation.

The change in wavelength or frequency of a monochromatic X-ray beam on scattering by matter is known as Compton effect and the difference in wavelength is called Compton shift.

Assumptions :

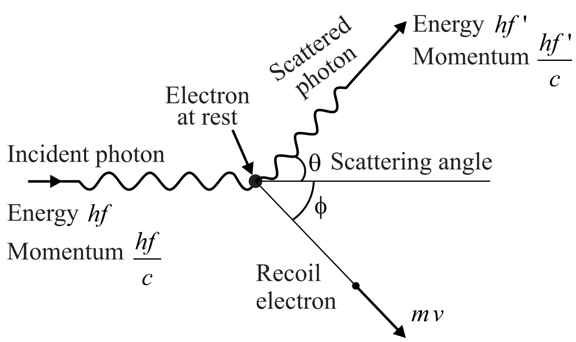
(i) X-ray radiation is quantized and may be regarded as a stream of photons. A single photon carries an energy hf. The photons behave like true particles and have momentum like any mechanical particle.

(ii) The electron in the atom of the target material is loosely bound and may be considered at rest initially.

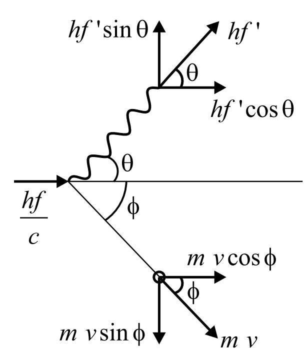
(iii) The phenomenon of scattering is due to an elastic collision between two particles, the photon of incident radiation and the electron scatterer.

Compton effect theory :

During the collision it gives a fraction of energy to the free electron. The electron gains kinetic energy and recoils. The process of recoiling of electron and scattering of photons is shown in Fig.1 In figure, θ is the scattering angle while ϕ is the recoil angle.



(a) Geometry of Compton scattering



(b) Components of momentum before and after collision

Fig.1

Before collision :

(i) Energy of incident photon $= hf$

(ii) Momentum of incident photon $= \frac{hf}{c}$

(iii) Rest energy of the electron $= m_0 c^2$, where m_0 is rest mass of the electron.

(iv) Momentum of rest electron $= 0$

After collision :

(i) Energy of scattered photon $= hf'$

(ii) Momentum of scattered photon $= \dfrac{hf'}{c}$

(iii) Energy of the electron $= mc^2$, where m is the mass of the electron moving with velocity v.

(iv) Momentum of recoil electron $= mv$, where v is the velocity of electron after collision.

and $$m = \frac{m_0}{\sqrt{\left[\left(1-\dfrac{v^2}{c^2}\right)\right]}} \qquad \text{...(i)}$$

Energy of the system (photon-electron) before collision $= hf + m_0 c^2$

Energy of the system after collision $= hf' + mc^2$

According to the principle of conservation of energy,

Energy before collision = Energy after collision

$$hf + m_0 c^2 = hf' + mc^2 \qquad \text{...(ii)}$$

Again, using the principle of conservation of momentum along and perpendicular to the direction of incident, we get

Momentum before collision = Momentum after collision

$$\frac{hf}{c} + 0 = \frac{hf'}{c}\cos\theta + mv\cos\phi \qquad \text{...(iii)}$$

In the perpendicular direction,

$$0 + 0 = \frac{hf'}{c}\sin\theta - mv\sin\phi \qquad \text{...(iv)}$$

From equation (iii),

$$mvc\cos\phi = hf - hf'\cos\theta \qquad \text{...(v)}$$

From equation (iv),

$$mvc\sin\phi = hf'\sin\theta \qquad \text{...(vi)}$$

Squaring equations (iv) and (v) and then adding, we get

$$(\cos^2\phi + \sin^2\phi)m^2v^2c^2 = (hf - hf'\cos\theta)^2 + (hf'\sin\theta)^2$$

$$m^2v^2c^2 = h^2f^2 - 2h^2 f f'\cos\theta + h^2 f'^2\cos^2\theta + h^2 f'^2\sin^2\theta$$

$$m^2v^2c^2 = h^2f^2 - 2h^2 ff'\cos\theta + h^2 f'^2(\cos^2\theta + \sin^2\theta)$$

$$m^2v^2c^2 = h^2[f^2 + f'^2 - 2ff'\cos\theta] \qquad \text{...(vii)}$$

From equation (ii), we get

$$mc^2 = h(f - f') + m_0 c^2$$

Squaring $m^2c^4 = h^2(f^2 - 2ff' + f'^2) + 2h(f - f')m_0c^2 + m_0^2c^4$ ...(viii)

Subtracting equation (vii) from equation (viii), we have

$$m^2c^4 - m^2v^2c^2 = -2h^2 ff'(1-\cos\theta) + 2h(f - f')m_0c^2 + m_0^2c^4$$

$$m^2c^2(c^2 - v^2) = -2h^2 ff'(1-\cos\theta) + 2h(f - f')m_0c^2 + m_0^2c^4$$

From equation (i) putting the value of m, we get

$$\frac{m_0^2c^2}{1-\dfrac{v^2}{c^2}}(c^2 - v^2) = -2h^2 ff'(1-\cos\theta) + 2h(f - f')m_0c^2 + m_0^2c^4$$

i.e., $$m_0^2c^4 = -2h^2 ff'(1-\cos\theta) + 2h(f - f')m_0c^2 + m_0^2c^4$$

$\therefore$ $$2h(f - f')m_0c^2 = 2h^2 ff'(1-\cos\theta) \qquad \text{...(ix)}$$

or $$\frac{f-f'}{ff'} = \frac{h}{m_0 c^2}(1-\cos\theta)$$

or $$\frac{1}{f'} - \frac{1}{f} = \frac{h}{m_0 c^2}(1-\cos\theta) \qquad \ldots(x)$$

Equation (x) shows that $f' < f$ as h, m_0, c are the constants with positive values and the maximum value of $\cos\theta = 1$. This shows that the scattered frequency is always smaller than the incident frequency.

From equation (x), we have

$$\frac{c}{f'} - \frac{c}{f} = \frac{h}{m_0 c}(1-\cos\theta)$$

or $$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta) \qquad \ldots(xi)$$

Where, $\Delta\lambda$ is change in wavelength.

The change in wavelength due to scattering is called as Compton effect.

From equation (xi) it is seen that,

(i) The wavelength λ' of the scattered photon is greater than the wavelength λ of the incident photon.

(ii) $\Delta\lambda$ is independent of the incident wavelength.

(iii) $\Delta\lambda$ has the same value for all substances containing free electrons. $\Delta\lambda$ only depends upon the angle of scattering.

(iv) When $\theta = 0, \cos\theta = 1$

$\therefore \quad \Delta\lambda = \lambda' - \lambda = 0$ or $\lambda' = \lambda$

which shows that no scattering occurs along the direction of incidence.

(v) When $\theta = \frac{\pi}{2}, \cos\theta = 0$

$$\text{Compton wavelength} = \Delta\lambda = \frac{h}{m_0 c} = \lambda_c$$

$$\lambda_c = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 0.0242\times10^{-10}\,\text{m} = 0.0242\,\text{Å}$$

(vi) As $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) = 0.0242(1-\cos\theta)\,\text{Å}$

Maximum value of

$\cos\theta = -1$ i.e., when $\theta = 180^0$

$\therefore \quad \Delta\lambda_{max} = 0.0484\,\text{Å}$

This means that Compton effect can be detected only for those radiations whose wavelength is not greater than a few Å.

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