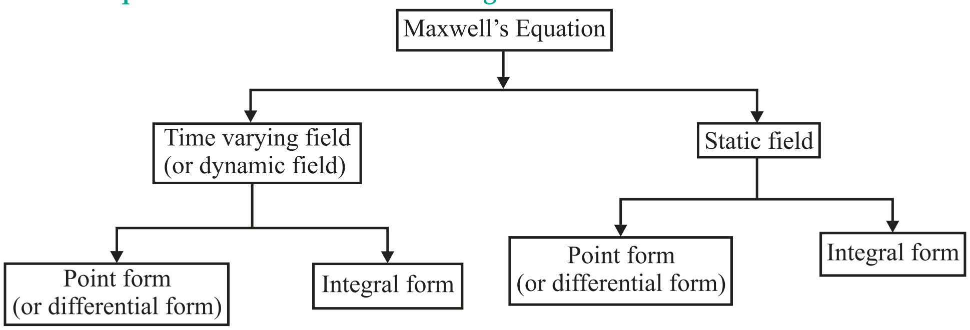
June : 2016 (CBCS)

Note : **Max. marks : 60**

Attempt any five questions. All questions carry equal marks.

Q.1 (a) Explain Maxwell's equations in differential and integral forms.

Ans. **Maxwell's equations in differential and integral forms :**



Maxwell's equations for dynamic Fields :

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_s \vec{D} \cdot d\vec{S} = \int_v \rho_v dv$	Gauss's law for electric field
$\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot d\vec{S} = 0$	Gauss's law for magnetic field
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} d\vec{S}$	Faraday's law of electromagnetic induction
$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J}_c + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$	Ampere's law

Maxwell's Equations for Static Fields :

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \vec{D} = \rho_v$	$\oint_s \vec{D} \cdot d\vec{S} = \int_v \rho_v dv$	Gauss's law
$\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot d\vec{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \vec{E} = 0$	$\oint_L \vec{E} \cdot d\vec{l} = 0$	Conservative nature of electrostatic field and Kirchoff's law
$\nabla \times \vec{H} = \vec{J}$	$\oint_L \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{S}$	Ampere's law

Q.1 (b) Obtain the divergence of following function.

$$\vec{f}(r) = 3x\hat{i} + 2y^2\hat{j} + 6z^3\hat{k}$$

Ans. **Given :** $\vec{f}(r) = 3x\hat{i} + 2y^2\hat{j} + 6z^3\hat{k}$

Formula :

The divergence of $\vec{f}$ is given by,

$$\nabla \cdot \vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \qquad \ldots(i)$$

Where, f_x, f_y, f_z = Component of vector $\vec{f}$ in the direction of x, y, z respectively.

Calculations :

From equation (i), we get

$$\nabla \cdot \vec{f} = \frac{\partial}{\partial x}(3x) + \frac{\partial}{\partial y}(2y^2) + \frac{\partial}{\partial z}(6z^3)$$

$$\nabla \cdot \vec{f} = 3 + 4y + 18z^2$$ **Ans.**

Q.2 (a) Explain the construction and working of a He-Ne laser with energy level diagram.

Ans. **He-Ne laser :** Helium Neon laser was the first successful gas laser. It is a four level laser.

He-Ne laser is a gas laser in which a mixture of He-Ne gases is used. The reason is that some of excited energy state of He are very close to excited state of Ne gas. He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics. Its usual operation wavelength is $6328\,\mathring{A}$, in the red portion of the visible spectrum.

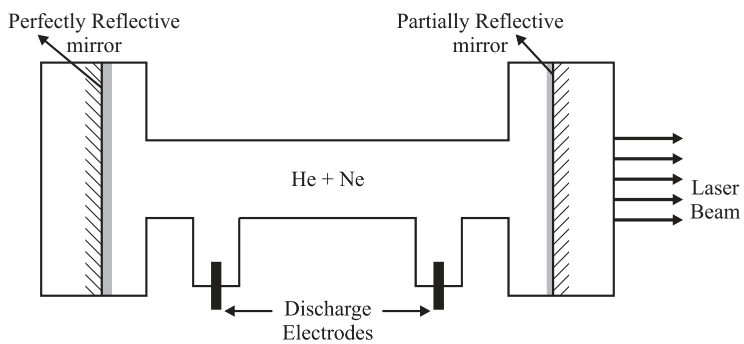


Fig.1 : He-Ne laser

Its main parts are :

1. **Working substances :** The active medium of He-Ne gas laser is a mixture of He and Ne gases in the ratio 10 : 1 at low pressure 1 Torr.
2. **Resonant cavity :** It is a discharge tube of length nearly 50 cm and diameter 1 cm with its ends perfectly plane and parallel. The left end of tube is a perfect reflector while right end is a partial reflector.
3. **Exciting source :** Population inversion is achieved by electric discharge.

Construction : It consists of a long discharge tube filled with He-Ne gases. One end of tube is provided with a perfect reflector and other end with a partial reflector. The pumping of gas mixture is done by electric discharge.

An electric discharge is produced in the gas by electrodes outside the tube connected to a source of high frequency alternating current.

Principle and working : The energy level diagram of He-Ne laser is shown in figure.

1. When the power is switched ON, the e^- from the discharge collide and pump the He and Ne atoms to metastable states 20.61 eV and 20.66 eV respectively above their ground state.

   Transition : $F_1 \rightarrow F_2$ Absorption.

2. Some of the excited He atom transfer their energy to Ne atom in collision with 0.05 eV of additional energy.

   Since the purpose of the He atom is to excite Ne atom and achieving a population inversion in the Ne-atom.

Energy transfer from, He $\rightarrow$ Ne .

3. When an excited Ne atom passes from metastable state at 20.66 eV to an excited state of 18.70 eV, it emits a photon of wavelength $6328\,\overset{\circ}{A}$. This photon travels through the gas mixture and if it is moving parallel to the axis of tube, is reflected back and forth by the mirror-ends until it stimulates an excited Ne atom and causes it to emit a $6328\,\overset{\circ}{A}$ photon in phase with stimulating photon.

The stimulated transition from 20.66 eV to 18.70 eV level is the laser transition.

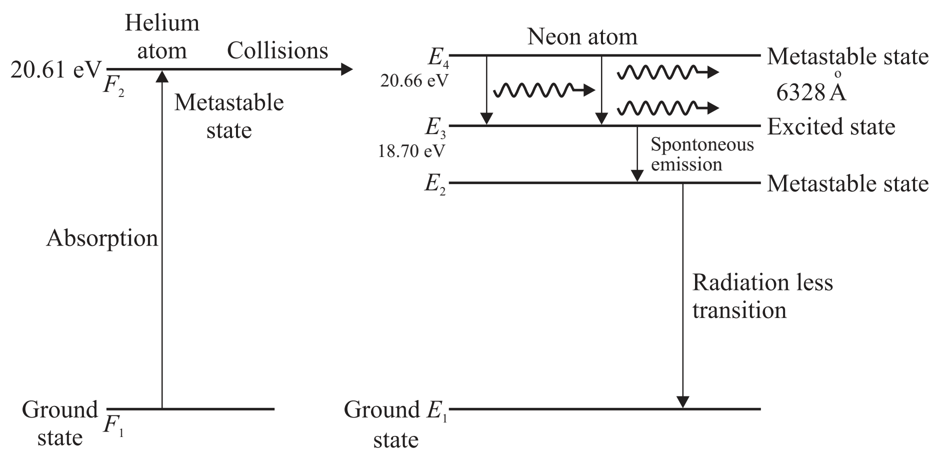


Fig.2 : Energy level diagram of He-Ne laser

The process is continued and a beam of coherent radiation becomes sufficiently intense.

Transition : $E_4 \rightarrow E_3$ Stimulated emission.

$20.66\,\text{eV} \rightarrow 18.70\,\text{eV}$ (Laser transition)

4. The Ne atom passes from the 18.70 eV level, spontaneously to a lower metastable state emitting incoherent light.

   Transition : $E_3 \rightarrow E_2$ Spontaneous emission.

5. Finally the Ne atom comes down to the ground state through collision with the tube walls. This transition, from lower metastable state E_2 to the ground state E_1 is radiation-less transition.

   Transition : $E_2 \rightarrow E_1$ radiation-less transition.

   Since the E_2 of neon is a metastable state then there is possibility of atoms in the level E_2 getting re-excited again to higher level E_3 . If this happens then it may disturb the population inversion at the level E_4 .

   This can be protected by reducing the diameter of the tube (i.e. narrow tube) so that atoms in the energy state E_2 follow the transition directly to the lower energy level E_1 mainly through the collision with the walls of tube instead of exciting atoms from the level E_2 to the level E_3 .

Q.2 (b) Give four major properties of a laser light.

Ans. **Characteristics of laser :** The abbreviation of laser is Light Amplification by Stimulated Emission of Radiation. Laser is a quantum electronics device which produces intense, monochromatic and coherent beam of light.

A laser beam has the following important characteristics :

1. **Directionality :** The laser emits light only in one direction while a conventional light source emits light in all direction. The width of laser beam is extremely narrow and hence a laser beam can travel to long distances without spreading.

2. **Monochromaticity :** It means that all the laser rays have same wavelength and frequency when they are emitted from the same source. Laser light is almost perfectly mono-chromatic.
3. **Coherence :** A conventional light source such as incandescent lamp or a natural source such as sun produces incoherent light since they emit random wavelength light waves with no common phase relationship. On the other hand, the waves emitted by a laser source will be in phase and are of same frequency. Therefore, light generated by a laser is highly coherent.
4. **Divergence :** Light from conventional sources spreads out in the form of spherical wave fronts and hence it is highly divergent. The divergence or angular spread of the laser beam is extremely small. The little divergence that exists in it arises out of the wave properties of light.

When the light issue out from the front mirror, it undergoes diffraction because the semitransparent mirror acts as a circular aperture. Accordingly, it spreads out and the angular spread is given by,

$$\lambda\theta = (1.22\lambda)/d$$

Where, d = Diameter the front mirror.

Q.3	(a)	Derive the expression for numerical aperture of a step index optical fiber.

Ans. **Numerical aperture :**

Fig. shows a light ray incident on the fibre core at angle θ_1 to the fibre axis which less than the acceptance angle for the fibre θ_a.

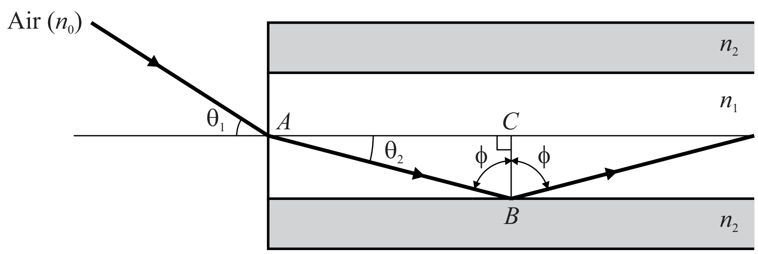


Fig. : The ray path for a meridional ray launched into an optical fibre in air at an input angle less than the acceptance angle to the fibre

The rays enters fibre from a medium (air) of refractive index n_0 and the fibre core has a refractive index n_1 which is slightly greater than the cladding refractive index n_2. Assuming the entrance face at the fibre core to be normal to the axis, then considering the refraction at the air-core interface and using Snell's law given by,

$$n_0 \sin\theta_1 = n_1 \sin\theta_2 \quad \text{...(i)}$$

Considering the right-angled triangle ABC indicated in figure, then

$$\phi = \frac{\pi}{2} - \theta_2 \quad \text{...(ii)}$$

Where, ϕ is greater than the critical angle at the core-cladding interface. Hence equation (i) becomes

$$n_0 \sin\theta_1 = n_1 \cos\phi \quad \text{...(iii)}$$

Using the trigonometric relationship $\sin^2\phi + \cos^2\phi = 1$, equation (iii) may be written as,

$$n_0 \sin\theta_1 = n_1 \left(1 - \sin^2\phi\right)^{1/2} \quad \text{...(iv)}$$

When the limiting case for total internal reflection is considered, ϕ becomes equal to the critical angle for the core-cladding interface and is given by,

$$\sin\phi_c = \frac{n_2}{n_1}$$

Also in this limiting case θ_1 becomes the acceptance angle for the fibre (θ_a). Combining these limiting case into equation (iv) gives,

$$n_0 \sin\theta_a = \left(n_1^2 - n_1^2\left(\frac{n_2^2}{n_1^2}\right)\right)^{1/2}$$

$$n_0 \sin\theta_a = \left(n_1^2 - n_2^2\right)^{1/2} \quad \ldots(v)$$

Equation (v), apart from relating the acceptance angle to the refractive indices, serves as the basis for the definition of the important optical fibre parameter, the numerical aperture (*N.A.*). Hence the *N.A.* is defined as,

$$N.A. = n_0 \sin\theta_a = \left(n_1^2 - n_2^2\right)^{1/2} \quad \ldots(vi)$$

Since the *N.A.* is often used with the fibre in air where n_0 is unity, it is simply equal to $\sin\theta_a$.

The *N.A.* may also be given in terms of the relative refractive index difference Δ between the core and the cladding which is defined as,

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$\Delta = \frac{n_1 - n_2}{n_1} \text{ for } \Delta << 1 \quad \ldots(vii)$$

Hence combining equation (vi) with equation (vii) we can write

$$N.A. = n_1\sqrt{2\Delta} \quad \ldots(viii)$$

Sometimes another parameter $\Delta n = n_1 - n_2$ is referred to as the index difference and $\Delta n / n_1$ as the fractional index difference. Hence Δ also approximates to the fractional index difference.

Q.3 (b) Obtain the *V*-number and number of modes supported by a step-index optical fiber having core index 1.48, cladding index 1.46 and the source wavelength 1.2 μm .

Ans. **Given :** Core refractive index $n_1 = 1.48$, Cladding refractive index $n_2 = 1.46$,

Wavelength $\lambda = 1.2\,\mu\text{m} = 1.2 \times 10^{-6}$ m .

Formula :

Numerical aperture is given by,

$$N.A. = \sqrt{n_1^2 - n_2^2} \quad \ldots(i)$$

Where, n_1 = Refractive index of core,

n_2 = Refractive index of clad.

Normalized frequency is given by,

$$V = \frac{\pi d}{\lambda}(N.A.) \quad \ldots(ii)$$

Where, d = Diameter,

$N.A.$ = Numerical aperture,

λ = Wavelength.

Number of modes is given by,

$$N = \frac{V^2}{2} \quad \ldots(iii)$$

Calculations :

From equation (i), we get

$$N.A. = \sqrt{1.48^2 - 1.46^2} = 0.2424$$

For a standard step index optical fibre, core diameter $= 50\mu$m.

From equation (ii), we get

$$V = \frac{\pi \times 50 \times 10^{-6}}{1.2 \times 10^{-6}} \times 0.2424 = 31.73$$ **Ans.**

From equation (iii), we get

$$N = \frac{31.73^2}{2} = 503$$ **Ans.**

Q.4 (a) Derive the expression for compton shift in a compton scattering process.

Ans. In 1992 Compton observed that when a monochromatic beam of X-ray was scattered by matter, the scattered beam contained radiation of two different wavelengths, one identical with the incident radiation and the other of longer wavelength.

The change in wavelength depends on the angle of observation.

Compton shift :

The change in wavelength or frequency of a monochromatic X-ray beam on scattering by matter is known as Compton effect and the difference in wavelength is called Compton shift.

Assumptions :

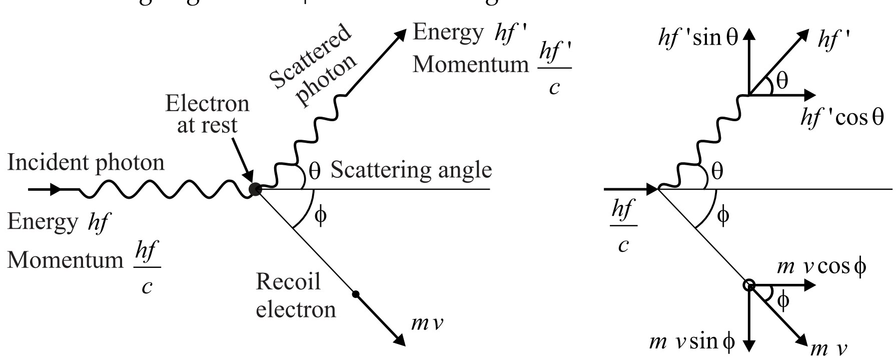
(i) X-ray radiation is quantized and may be regarded as a stream of photons. A single photon carries an energy hf. The photons behave like true particles and have momentum like any mechanical particle.

(ii) The electron in the atom of the target material is loosely bound and may be considered at rest initially.

(iii) The phenomenon of scattering is due to an elastic collision between two particles, the photon of incident radiation and the electron scatterer.

Compton effect theory :

During the collision it gives a fraction of energy to the free electron. The electron gains kinetic energy and recoils. The process of recoiling of electron and scattering of photons is shown in Fig.1 In figure, θ is the scattering angle while ϕ is the recoil angle.



(a) Geometry of Compton scattering

(b) Components of momentum before and after collision

Fig.1

Before collision :

(i) Energy of incident photon $= hf$

(ii) Momentum of incident photon $= \frac{hf}{c}$

(iii) Rest energy of the electron $= m_0 c^2$, where m_0 is rest mass of the electron.

(iv) Momentum of rest electron $= 0$

After collision :

(i) Energy of scattered photon $= hf'$

(ii) Momentum of scattered photon $= \frac{hf'}{c}$

(iii) Energy of the electron $= m c^2$, where m is the mass of the electron moving with velocity v.

(iv) Momentum of recoil electron $= m v$, where v is the velocity of electron after collision.

and $$m = \frac{m_0}{\sqrt{\left[\left(1-\frac{v^2}{c^2}\right)\right]}} \qquad \ldots(i)$$

Energy of the system (photon-electron) before collision $= hf + m_0 c^2$

Energy of the system after collision $= hf' + m c^2$

According to the principle of conservation of energy,

Energy before collision = Energy after collision

$$hf + m_0 c^2 = hf' + m c^2 \qquad \ldots(ii)$$

Again, using the principle of conservation of momentum along and perpendicular to the direction of incident, we get

Momentum before collision = Momentum after collision

$$\frac{hf}{c} + 0 = \frac{hf'}{c}\cos\theta + m v \cos\phi \qquad \ldots(iii)$$

In the perpendicular direction,

$$0 + 0 = \frac{hf'}{c}\sin\theta - m v \sin\phi \qquad \ldots(iv)$$

From equation (iii),

$$m v c \cos\phi = hf - hf' \cos\theta \qquad \ldots(v)$$

From equation (iv),

$$m v c \sin\phi = hf' \sin\theta \qquad \ldots(vi)$$

Squaring equations (iv) and (v) and then adding, we get

$$(\cos^2\phi + \sin^2\phi) m^2 v^2 c^2 = (hf - hf'\cos\theta)^2 + (hf'\sin\theta)^2$$

$$m^2 v^2 c^2 = h^2 f^2 - 2h^2 f f'\cos\theta + h^2 f'^2 \cos^2\theta + h^2 f'^2 \sin^2\theta$$

$$m^2 v^2 c^2 = h^2 f^2 - 2h^2 ff'\cos\theta + h^2 f'^2 (\cos^2\theta + \sin^2\theta)$$

$$m^2 v^2 c^2 = h^2 [f^2 + f'^2 - 2 ff'\cos\theta] \qquad \ldots(vii)$$

From equation (ii), we get

$$m c^2 = h(f - f') + m_0 c^2$$

Squaring $m^2 c^4 = h^2 (f^2 - 2 f f' + f'^2) + 2h(f - f') m_0 c^2 + m_0^2 c^4$

$\ldots$(viii)

Subtracting equation (vii) from equation (viii), we have

$$m^2 c^4 - m^2 v^2 c^2 = -2h^2 ff'(1-\cos\theta) + 2h(f - f') m_0 c^2 + m_0^2 c^4$$

$$m^2 c^2 (c^2 - v^2) = -2h^2 ff'(1-\cos\theta) + 2h(f - f') m_0 c^2 + m_0^2 c^4$$

From equation (i) putting the value of m, we get

$$\frac{m_0^2 c^2}{1-\frac{v^2}{c^2}}(c^2 - v^2) = -2h^2 f f'(1-\cos\theta) + 2h(f - f') m_0 c^2 + m_0^2 c^4$$

i.e., $$m_0^2 c^4 = -2h^2 ff'(1-\cos\theta) + 2h(f - f') m_0 c^2 + m_0^2 c^4$$

$\therefore$ $$2h(f - f') m_0 c^2 = 2h^2 ff'(1-\cos\theta) \qquad \ldots(ix)$$

or $$\frac{f - f'}{ff'} = \frac{h}{m_0 c^2}(1-\cos\theta)$$

or $$\frac{1}{f'} - \frac{1}{f} = \frac{h}{m_0 c^2}(1-\cos\theta) \qquad \ldots(x)$$

Equation (x) shows that $f' < f$ as h, m_0, c are the constants with positive values and the maximum value of $\cos\theta = 1$. This shows that the scattered frequency is always smaller than the incident frequency.

From equation (x), we have

$$\frac{c}{f'} - \frac{c}{f} = \frac{h}{m_0 c}(1-\cos\theta)$$

or $$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta) \qquad \text{...(xi)}$$

Where, $\Delta\lambda$ is change in wavelength.

The change in wavelength due to scattering is called as Compton effect.

From equation (xi) it is seen that,

(i) The wavelength λ' of the scattered photon is greater than the wavelength λ of the incident photon.

(ii) $\Delta\lambda$ is independent of the incident wavelength.

(iii) $\Delta\lambda$ has the same value for all substances containing free electrons. $\Delta\lambda$ only depends upon the angle of scattering.

(iv) When $\theta = 0, \cos\theta = 1$

$$\therefore \quad \Delta\lambda = \lambda' - \lambda = 0 \text{ or } \lambda' = \lambda$$

which shows that no scattering occurs along the direction of incidence.

(v) When $\theta = \frac{\pi}{2}, \cos\theta = 0$

Compton wavelength $= \Delta\lambda = \frac{h}{m_0 c} = \lambda_c$

$$\lambda_c = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 0.0242\times10^{-10}\,\text{m} = 0.0242\,\text{Å}$$

(vi) As $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) = 0.0242(1-\cos\theta)\,\text{Å}$

Maximum value of

$$\cos\theta = -1 \text{ i.e., when } \theta = 180^0$$

$$\therefore \quad \Delta\lambda_{max} = 0.0484\,\text{Å}$$

This means that Compton effect can be detected only for those radiations whose wavelength is not greater than a few Å.

Q.4 (b) Explain Heisenberg Uncertainty principle.

Ans. **Uncertainty principle :** In 1927, German physicist Werner Heisenberg provided an interesting addition to the meaning of the wave particle concept. He stated a very important principle, known as the uncertainty principle.

The principle can be stated as follows :

"It is impossible to determine the exact position and momentum of a particle simultaneously."

The uncertainty principle is expressed by the relation

$$\Delta x \cdot \Delta p \geq \hbar$$

When we consider a group consisting of very large number of harmonic waves of continuously varying frequencies, the product of the uncertainties comes to

$$\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$$

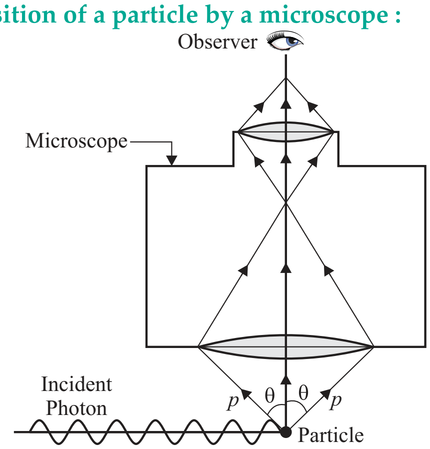
Where, $\hbar = \frac{h}{2\pi}$

$\Delta x =$ Uncertainty in determining the position of particle,

Δp = Uncertainty in determining the momentum of particle.

The various applications of uncertainty principle are :

1. Determination of position of a particle by a microscope :



Consider the case of the measurement of the position of a particle say electron in the field of microscope. The resolving power i.e., the smallest distance between the two points that can be just resolved by the microscope is given by,

$$\Delta x \approx \frac{\lambda}{2\sin\theta} \quad \text{...(i)}$$

Where λ the wavelength of light is used, θ the semi-vertical angle of the cone of light and Δx is the uncertainty in determining the position of the particle.

In order to observe the electron it is necessary that at least one photon must strike the electron and scatter inside the microscope. When a photon of initial momentum $p = h/\lambda$, after scattering enters the field of view of microscope, it may be anywhere within angle 2θ. Thus its x component of momentum i.e., p_x may lie between $p\sin\theta$ and $-p\sin\theta$. As the momentum is conserved in the collision, the uncertainty in the x component of momentum is given by,

$$\Delta p_x = p\sin\theta - (-p\sin\theta)$$

$$\Delta p_x = 2p\sin\theta$$

$$\Delta p_x = 2\frac{h}{\lambda}\sin\theta \qquad \left[\because p = \frac{h}{\lambda}\right] \quad \text{...(ii)}$$

From equations (i) and (ii), we have

$$\Delta x \cdot \Delta p_x = \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta \approx h > \frac{\hbar}{2}$$

This shows that the product of uncertainties in position and momentum is of the order of Planck's constant.

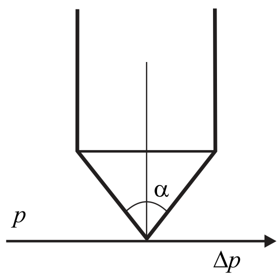
For macroscopic object the uncertainty principle has no importance because for such object, the position and momentum could always be determined simultaneously with absolute accuracy.

2. Diffraction of an electron beam by a single slit :

Let us consider a narrow beam of electrons of momentum p passing through a narrow slit of width Δy [shown in Fig.2], Δy is a measure of uncertainty in the position of electron. Diffraction pattern will appear on the screen as shown in Fig.1. If we assume that the screen is far away relative to the width of the slit, the first minimum of the Fraunhofer diffraction pattern is obtained by putting $n = 1$ in the equation ($d\sin\alpha = n\lambda$) describing the diffraction pattern due to single slit. Thus,

$$\Delta y \sin\alpha = \lambda$$

$$\Delta y = \frac{\lambda}{\sin\alpha} \quad \text{...(iii)}$$



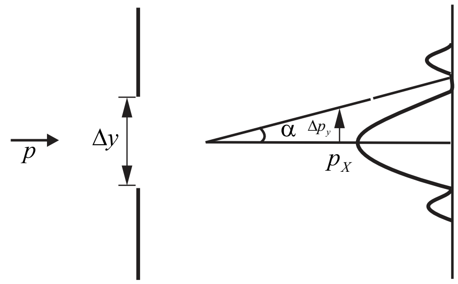


Fig.1 **Fig.2**

The moving electrons in the beginning have no component of momentum along y-axis, since they are moving along x-axis. But at the slit they deviate from their original path to form a pattern on the screen and hence have a component of momentum $p\sin\alpha$ in the y-direction. Now as the electron may be anywhere within the pattern from angle $-\alpha$ to $+\alpha$, the y-component of momentum of electron may lie anywhere between $p\sin\alpha$ and $-p\sin\alpha$. Obviously, the uncertainty in the y-component of the momentum of electron is,

$$\Delta p = p\sin\alpha - (-p\sin\alpha) = 2p\sin\alpha$$

$$\Delta p = \frac{2h}{\lambda}\sin\alpha \quad \text{...(iv)}$$

From equation (iii) and (iv), we have

$$\Delta x \cdot \Delta p = \frac{\lambda}{\sin\alpha} \frac{2h}{\lambda}\sin\alpha = 2h$$

which is in agreement with the uncertainty principle.

3. **Non-existence of electrons in the nucleus :** The radius of the nucleus is of order $\approx 10^{-14}$ m. If any type of particle is to exist in the nucleus, the uncertainty in their position in the nucleus is given by,

$$\Delta x = 2 \times 10^{-14}\,\text{m}$$

$\therefore$ The uncertainty in the momentum of the particle in the nucleus is given by,

$$\Delta p_x = \frac{h}{2\pi\Delta x} = \frac{6.62 \times 10^{-34}}{2\pi \times 2 \times 10^{-14}\,\text{m}} = 5.2 \times 10^{-21}\,\text{kg m/sec}$$

Thus, the magnitude of the momentum of the particle in the nucleus must be at least of this order.

$\therefore$ The relativistic energy of an electron in the nucleus is given by,

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$E = \sqrt{(9.1 \times 10^{-31})^2 (3 \times 10^8)^4 + (5.2 \times 10^{-21})^2 \times (3 \times 10^8)^2}$$

$$E = \sqrt{(6.7 \times 10^{-27}) + (2.4 \times 10^{-24})}$$

$$E = 1.56 \times 10^{-12} \text{ Joules} \qquad [1\,\text{eV} = 1.6 \times 10^{-19}\text{ J}]$$

$$E = 9.7\,\text{MeV}$$

Thus, if the electron resides in the nucleus, it should have an energy of the order of $10\,\text{MeV}$. However, electrons emitted during β-decay have energies of the order of $3\,\text{MeV}$ to $4\,\text{MeV}$, which varies widely from calculated value. Hence we conclude that electrons cannot reside in the nucleus.

Q.5 (a) Derive the condition for maxima and minima in interference of light reflected from a thin film.

Ans. Consider a parallel-side transparent thin film of thickness t and refractive index $\mu(>1)$. A ray of monochromatic light SA incident at an angle i is partly reflected along AR_1 and partly refracted along AB at angle r. At B it is again partly reflected along BC and partly refracted along BT_1, as shown in figure. Similar reflections and refractions occur at C, D,…..etc.

Reflected system : Now let us consider the reflected rays only. At points A, B, C ….only a small part of light is reflected, the remaining portion is refracted.

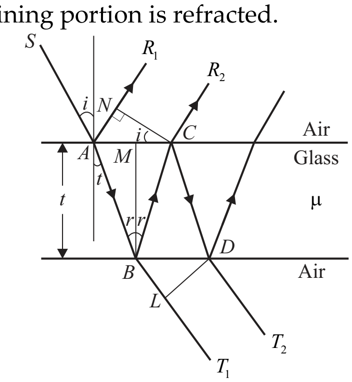


Fig. : Interference in thin films

Therefore, the rays AR_1 and CR_2 each having undergone one reflection, have almost equal intensities. Rest of the rays can be ignored. Hence AR_1 and CR_2 rays are in a position to interfere.

So first, we will calculate the path difference between rays AR_1 and CR_2. For this, draw CN and BM perpendicular to AR_1 and AC then,

Path difference, Δ = Path ABC in film – Path AN in air

$$\Delta = \mu(AB + BC) - AN \qquad \text{…(a)}$$

In right angled triangle ABM,

$$\cos r = \frac{BM}{AB}$$

$$AB = \frac{BM}{\cos r} = \frac{t}{\cos r} \qquad [\because BM = t]$$

Where, t = Thickness of film and $\angle ABM = r$.

and
$$AN = AC \sin i$$

$$AN = (AM + MC)\sin i = (BM \tan r + BM \tan r)\sin i$$

$$AN = 2t \tan r \sin i$$

$$AN = 2t \cdot \frac{\sin r}{\cos r} \sin i$$

$$AN = 2t \frac{\sin r}{\cos r} \mu \sin r = 2\mu t \frac{\sin^2 r}{\cos r} \qquad [\because \sin i = \mu \sin r]$$

Therefore, using these values, path difference becomes, Path difference, $\Delta = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$ [From equation (a)]

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$\therefore$ Path difference = $2\mu t \cos r$ ...(i)

Since the ray AR_1 suffers reflection at the surface of a denser medium, therefore, it undergoes a phase change of π (or path difference of $\lambda/2$).

Hence the effective path difference between rays AR_1 and CR_2 is,

$$= 2\mu t \cos r + \frac{\lambda}{2} \quad \text{...(ii)}$$

Conditions of maxima and minima in reflected light :

The two rays will produce constructive interference if the path difference between them is an integral multiple of λ.

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2} = n\lambda \quad \text{When, } n = 0, 1, 2....$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \quad \text{(Condition of maxima)} \quad \text{...(iii)}$$

When this condition is satisfied, then the film will appear **bright in the reflected light.**

The two rays will produce destructive interference if the path difference between them is an odd multiple of $\frac{\lambda}{2}$,

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{When, } n = 0, 1, 2.......$$

$$2\mu t \cos r = n\lambda \quad \text{(Condition of minima)} \quad \text{...(iv)}$$

When this condition is satisfied, then the film will appear **dark in the reflected light.**

Q.5 (b) Enlist four differences between interference and diffraction.

Ans. Difference between interference and diffraction :

S. No.	Interference	Diffraction
1.	This phenomenon is the result of interaction taking place between two separate wavefronts originating from two coherent sources.	This phenomenon is the result of interaction of light between the secondary wavelets originating from different points of same wave front.
2.	The region of minimum intensity are usually almost perfectly dark.	The region of minimum intensity are not perfectly dark.
3.	Interference fringes may or may not be of the same width.	Diffraction fringes are not of the same width.
4.	All maxima are of same intensity.	Maxima are of varying intensity.

Q.6 (a) Explain Hall effect and derive expression for Hall mobility.

Ans. **Hall effect :** When a metal or a semiconductor carrying a current I is placed in a transverse magnetic field B, a potential difference is produced in the direction normal to both the current and magnetic field directions. This phenomenon is called **Hall effect** and potential difference developed is called **Hall voltage**.

Hall effect measurements showed that it is the negative charge carriers namely electrons which are responsible for electrical conduction in metals. It also showed that there exist two types of charge carriers in semiconductors.

Importance : The importance of Hall effect is that it helps to determine the :

(i) Sign of charge carriers.

(ii) Charge carriers concentration.

(iii) Mobility of charge carriers if conductivity of the material is known.

(iv) Type of semiconductor.

Experimental determination of Hall voltage and Hall coefficient :

Let us consider an n-type semiconductor in which the conduction is predominated by electrons. Suppose an electric current I flows in the positive x-direction and a magnetic field B is applied normal to this electric field in z-direction as shown in Fig. (a). A force, called the Lorentz force is exerted on each electron which causes the electron paths to bend. As a result of this, the electrons accumulate on one side of the slab and are deficient on the other side.

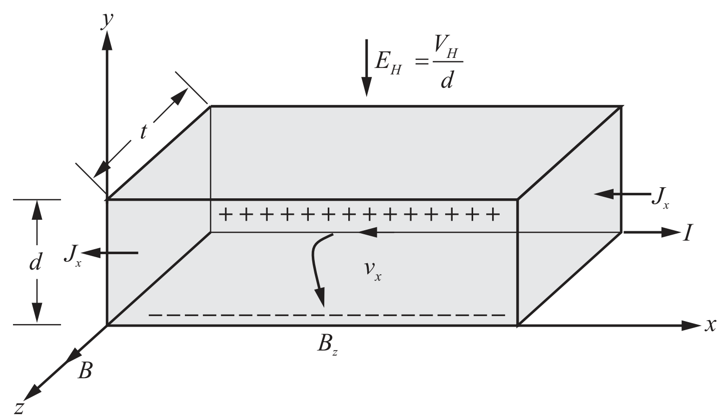


Fig. (a) Schematic view of an n-type semiconductor bar.

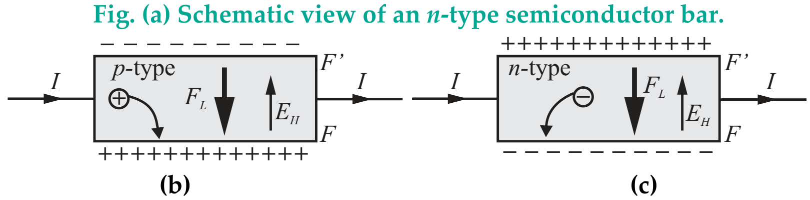


Fig. The direction of magnetic force and Hall field (b) in p-type semiconductor (c) in an n-type semiconductor

Thus, an electric field is created in the y-direction which is called the **Hall field**. In equilibrium condition,

Hall force = Lorentz force

$$F_H = F_L$$

$$-qE_H = v_x B_z q \qquad \ldots\text{(i)}$$

Where, v_x is the velocity of the electrons, and q the electronic charge.

$$\therefore \qquad E_H = v_x B_z \qquad \ldots\text{(ii)}$$

As current density, $J_x = -Nv_x q$

Putting the value of v_x from equation (ii) in the above equation, we get

$$J_x = -\frac{NE_H q}{B_z}$$

$$N = \frac{-J_x B_z}{qE_H} = -\frac{J_x B_z}{q(V_H / d)} \quad \ldots(iii)$$

$$V_H = -\frac{J_x B_z d}{Nq} = -\frac{IB_z d}{NqA} \quad \ldots(iv)$$

Where, A is the area of cross-section of end face.

If t is the thickness of the semiconductor specimen, $A = dt$ and the above equation reduces to,

$$V_H = -\frac{B_z I}{Nqt} \quad \ldots(v)$$

Hall field per unit current density per unit magnetic induction is called **Hall coefficient** R_H. Thus,

$$R_H = \frac{E_H}{J_x B_z} = +\frac{V_H / d}{J_x B_z} = -\frac{B_z I}{J_x B_z dNqt}$$

$$R_H = -\frac{1}{Nq} = \frac{1}{\rho} \quad \ldots(vi)$$

Where, ρ is the charge density.

In terms of Hall coefficient, Hall voltage is given by,

$$V_H = R_H \frac{BI}{t} \quad \ldots(vii)$$

The sign of the Hall coefficient R_H indicates whether electrons or holes predominate in the conduction process.

Experimental determination of mobility :

The electron mobility is given as,

$$\mu_n = \frac{\sigma}{N|q|}$$

Hence, $\mu_n = |R_H|\sigma$

Thus, the magnitude of μ_n can be determined if the conductivity σ has been measured.

Hall angle :

The net electric field E in the semiconductor is a vector sum of E_x (electric field component in x-direction) and E_H (Hall field). It acts at an angle θ_H to the x-axis. θ_H is called the **Hall angle**.

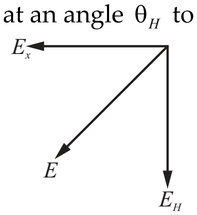


Fig. (d)

From Fig. (d),

$$\tan\theta_H = \frac{E_H}{E_x} \quad \ldots(viii)$$

As $E_H = \frac{V_H}{d} = \frac{B_z J_x}{N|q|}$ ...(ix)

Also $E_x = \rho J_x$ ...(x)

$\therefore \qquad \tan\theta_H = \dfrac{B_z}{N|q|\rho} \qquad \left[\because \sigma = \dfrac{1}{\rho}\right]$

$\therefore \qquad \tan\ \theta_H = \sigma R_H B_z$

The product σR_H is designated as μ_n the mobility of electrons.

$\therefore \qquad \tan\ \theta_H = \mu_n B_z$

$\therefore \qquad \theta_H = \tan^{-1}(\mu_n B_z)$ …(xi)

In the above discussions it is assumed that all carriers travel with a mean speed v_x. However, this does not happen. As a result the value of R_H gets modified. The appropriate value is

$$R_H = \frac{3\pi}{8}\left(-\frac{1}{Nq}\right) \qquad \text{…(xii)}$$

Accordingly,

$$\mu_n = \frac{8}{3\pi}\sigma R_H \qquad \text{…(xiii)}$$

Applications :

1. **Magnetic field meter :** The Hall voltage V_H for a given current is proportional to *B*. Hence measurement of V_H measures the magnetic field *B*.
2. **Hall effect multiplier :** This instrument gives an output proportional to the product of two signals. Thus, if current *I* is made proportional to one input and if *B* is proportional to the second input, then Hall voltage V_H is proportional to the product of the two inputs.

Q.6 (b) Explain *V-I* characteristics of a photovoltaic cell.

Ans. **Photovoltaic cell :** A solar cell is nothing but a *p-n* junction device based on the principle of photoelectric effect. It directly converts light into electricity and, hence, is known as a **photovoltaic cell.**

Description : Conventional silicon cells are thin wafers about $300\,\mu m$ thick 3 to 6 cm in diameter, sliced from a single crystal of *n*-type or *p*-type doped silicon as shown in Fig.1.

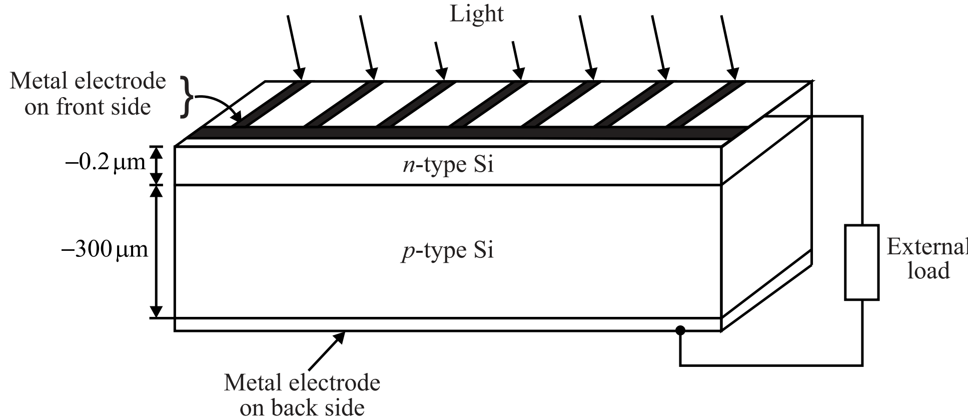


Fig.1 : Solar cell

A shallow junction is formed at one end by the diffusion of the other type of impurity. Metal electrodes are made of Ti-Ag solder and are attached to the front and back sides. The electrodes are in the form of a metal grid with figures which permit sunlight to pass through it. On the back side, the electrode completely covers the surface. An antireflection coating of SiO having a thickness of about $0.1\ \mu m$, is also put on the top surface.

***V-I* characteristics of a solar cell :**

When radiation falls on a solar cell, it is absorbed and pairs of positive and negative charges, celled electron-hole pairs are created. The positive and negative charges are separated because of the *p-n* junction. The direct current thereby produced is collected by the metal electrodes and flows through the external load.

The volt-ampere characteristics of a solar cell are determined by connecting a decade resistance box and a voltmeter as shown in Fig.2.

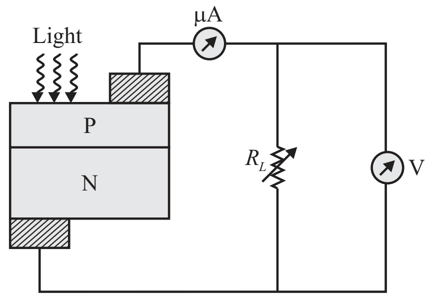


Fig.2 : *V-I* characteristics of a solar cell

A known intensity of light from a halogen is made to fall on it. The resistance values are varied step by step and the corresponding voltages across the resistance box is measured using a voltmeter. From the known values of *V* and *R*, the value of *I* is determined using the relation, *I* = *V*/*R*. A graph is drawn between voltage and current is shown in Fig.3.

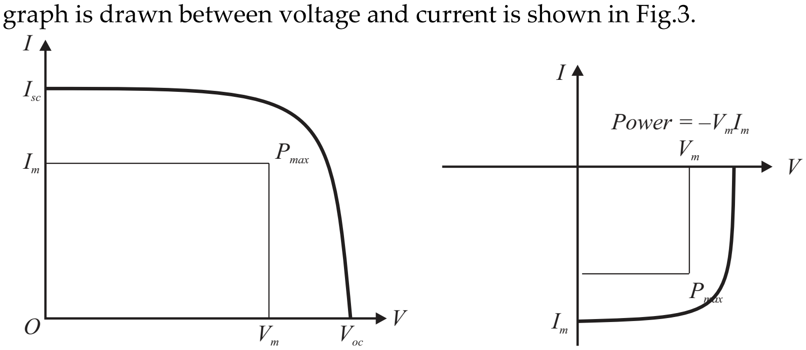


Fig.3 : Characteristics curve

The current I_{sc} shown in Fig. 3 is obtained by short-circuit in the two terminals of the solar cell and this current is known as short-circuit current. Similarly, voltage V_{oc} is known as open-circuit voltage. The product of these two quantities gives the ideal power of the cell. The maximum useful power is the area of the largest rectangle that can be formed in the *V-I* curve. The corresponding voltage and current are represented by $V_m I_m$. The ratio of the maximum useful power to the ideal power is called the fill factor. A typical value of the V_{oc}, I_{sc} and fill factor for a silicon cell are respectively, 450 to 600 mV, 30 to 50 mA and 0.65 to 0.8.

Q.7 (a) Explain the liquid drop model for a nucleus and various energy terms therein.

Ans. **Nuclear liquid drop model :** This model was proposed by Neils Bohr in 1936. He pointed out the strong, short-range, attractive forces between the nucleons that are analogous to those acting between the molecules of liquid. The surface molecules in a liquid drop are attracted by the inner molecules thus generating the surface tension. This surface tension helps the liquid drop to take spherical shape. He proposed almost similar phenomenon to nucleus. The two important properties of a nucleus, the constant density and the constant binding energy per nucleon can be matched with the liquid drop. Thus on the basis of Bohr's liquid drop model the following conclusions may be drawn.

The similarities between the nucleus and a liquid drop are as follows :

1. In the stable state, the nucleus in shape, just like as liquid drop is spherical due to the symmetrical surface tension forces.

2. The force of surface tension acts on the surface of the liquid drop. Similarly there is a potential barrier at the surface of the nucleus.
3. The density of a liquid drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.
4. The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiation almost immediately.
5. The nuclear forces are short range forces, similarly, as that of liquid, in which the intermolecular forces are short range forces.
6. When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar in which the nucleus break up into two smaller nuclei.

Q.7 (b) Differentiate between nuclear fission and fusion processes.

Ans. Difference between nuclear fission and fusion :

S. No.	Nuclear fission	Nuclear fusion
1.	Nuclear fission is a process in which a heavy nucleus, after capturing a neutron splits up into two lighter nuclei of comparable masses. **Example :** ${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_{0}n^{1} + Q$	Nuclear fusion is a process in which two lighter nuclei combine to produce a heavy and stable nucleus. **Example :** $4{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2{}_{1}\beta^{0} + Q$
2.	The process is possible even at room temperature.	The process is possible only at very high temperature.
3.	The links of the process are neutrons.	The links of the process are protons.
4.	Energy released by the fission of one nucleus of U^{235} is about 200 MeV.	Energy released by the fusion is about 26.7 MeV.
5.	Energy released per nucleon is 0.85 MeV.	Energy released per nucleon is 6.67 MeV.
6.	The fissionable substance is radioactive.	The fusionable substance is not radioactive.

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